CONCRETE

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AND

CONSTRUCTIONAL ENGINEERING

INCLUDING PRESTRESSED CONCRETE

MAY 1958 **VOL. LIII. NO. 5**

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LEADING CONTENTS

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Volume LIII, No. 5. LONDON, MAY, 1958.

EDITORIAL NOTES

New British Standards for Portland Cements.

THE value of cement as a binder was until recent years judged only by its tensile strength. This had little justification, for it is seldom that a construction is subjected to a direct pull such as is used in tensile tests of mortar briquettes. For this reason in some countries the tensile test has been replaced by bending tests in which bars of mortar are tested as beams, and it is interesting to note that an optional bending test is included in the revision of B.S. Code of Practice for Reinforced Concrete issued a few months ago. In the new British Standards issued last month for ordinary and rapid-hardening Portland cements, Portland blastfurnace cement, and low-heat Portland cement a bending test is not included and the tensile test has been abandoned except that in the case of rapid-hardening cement the customer may call for a tensile test at one day. It is, however, not likely that the desirability of increasing the tensile strength of cement will be overlooked, for greater tensile strength could lead to economies. The tensile strength of concrete is so small that it is generally ignored in the design of reinforced concrete structures, but a considerable increase in the tensile strength of concrete would often result in a saving of the amount of steel necessary to resist tensile and shearing forces. Greater tensile strength would also often permit economies to be made in precast concrete in which reinforcement is often used only to resist tensile stresses during transport and erection, and in unreinforced products such as tiles and kerbs for which a minimum strength in bending is required by British Standards.

In the new B.S. No. 12 the compressive strength of 1 : 3 mortar cubes made with ordinary Portland cement is increased from 1600 lb. to 2200 lb. per square inch at three days and from 2500 lb. to 3400 lb. at seven days. In the previous standard of 1947 the required minimum compressive strengths of 1 : 3 mortar cubes made with rapid-hardening Portland cement were 1600 lb. per square inch at one day and 3500 lb. at three days. In the new standard the one-day test is omitted, and the requirements are 3000 lb. per square inch at three days and 4000 lb. at seven days. The strength now required at three days is the same as in the U.S.A. standard of 1955.

The most interesting feature of the new standards is the inclusion of a method of assessing the quality of cement by a compressive test of concrete. In compressive tests the specimens fail by shearing rather than by actual crushing; such a test is therefore also a measure of the tensile strength of the material,

because the tensile strength varies generally with the compressive strength, as is the case with natural stones. The cubes for testing are to have 4-in. sides, and are to be made with graded stone up to $\frac{3}{4}$ in. and sand up to $\frac{3}{16}$ in. The use of compressive tests of concrete to assess the quality of cement was devised some years ago in the laboratory of the Associated Portland Cement Manufacturers, Ltd.; it has since been recommended by a committee representing twelve countries, but Great Britain is the first to incorporate this test in a national standard for cement. In such a test the properties of the aggregate, which comprises the greater part of the specimen, are important if the results are to be comparable. Much thought was given to the use of standard gravels and sands, but it was considered desirable that it should be possible to make the test with the aggregates to be used in the work. Tests showed that if flint, granite, limestone, porphyry, or quartzite conforming to B.S. No. 882, "Concrete Aggregates from Natural Sources", were used the variation in the results of tests made in different laboratories did not exceed 10 per cent. Any coarse and fine aggregates conforming to B.S. No. 882 may therefore be used, and the expense of obtaining a standard aggregate from one source is avoided. Only the quantities of cement and water and a slump of $\frac{1}{2}$ in. to 2 in. are specified, and these result in proportions of about 1 : 2 : 4 by weight and a water-cement ratio of 0·6.

The requirements are 1200 lb. per square inch at three days and 2000 lb. at seven days in the case of ordinary Portland cement, and 1700 lb. at three days and 2500 lb. at seven days for rapid-hardening Portland cement. These minima should be much exceeded with British cements, and may be compared with the minimum compressive strength of 2700 lb. per square inch at seven days recommended for preliminary tests by the new B.S. Code of Practice for Reinforced Concrete. It is almost certain that 1 : 2 : 4 concrete with a water-cement ratio of 0·6 will meet the requirement of the Code; if, however, the strength is only a little more than 2000 lb. as required by the new standard for Portland cement, and it is not possible to increase the strength by reducing the water-cement ratio, advantage may be taken of Clause 208 (b) of the Code which permits the use of such concrete with a cube strength in preliminary tests of 4000 lb. at 28 days.

Among other changes in the standard for ordinary and rapid-hardening Portland cements are the omission of the test for fineness by limiting the residue on a 170-mesh sieve; specific surface areas only are now used as a criterion of fineness, and these are not altered. The initial setting period is increased from 30 minutes to 45 minutes; the final setting time is unchanged at ten hours.

In the case of Portland-blastfurnace cement the tensile test is omitted, the compressive strength of mortar cubes remains at 1600 lb. per square inch at three days and at seven days is increased to 3000 lb. per square inch; an optional test at 28 days, when the strength must be 5000 lb. per square inch, is included. Tests on concrete cubes are included, the compressive strengths required being 800 lb. per square inch at three days, 1600 lb. at seven days, and an optional test (3200 lb. per square inch) at 28 days. In the case of low-heat cements the compressive strengths of mortar cubes are increased from 1000 lb. to 1100 lb. per square inch at three days, from 1600 lb. to 2000 lb. at seven days, and from 3750 lb. to 4000 lb. at 28 days. Compressive tests on concrete cubes are also included, and the strengths required are 500 lb. per square inch at three days, 1000 lb. at seven days, and 2000 lb. at 28 days.

Beams and Slabs Designed by the Load-factor Method.

In Accordance with the B.S. Code of 1957.

By CHAS. E. REYNOLDS, B.Sc.(Eng.), A.M.I.C.E.

THE load-factor method of designing beams and solid slabs recommended in B.S. Code No. 114 (1957) is simple to apply, and differs from most other published methods in so far as the formulæ are in terms of the permissible working stresses instead of the crushing strength of the concrete and the yield stress of the steel. The formulæ * take these properties into account but factors of safety are introduced to enable the calculations to be prepared in a manner similar to the ordinary modular-ratio method (elastic theory) in terms of the applied bending moment and the permissible stresses. In the following the basic formulæ given in the Code are modified to enable a more direct application to design to be made, and tables are given where their use makes a worth-while saving of arithmetical labour. The notation, which is the same as in the Code with such additional symbols as are required in the present treatment, is as follows:

A_{sc}, A_{st}, areas of compressive and tensile reinforcement respectively; A_{sc}', A_{st}', do. in unit width of a rectangular beam.

a_1, lever-arm factor $= l_a/d_1$.

b, breadth of a rectangular beam; breadth of flange of a flanged beam; b_r, breadth of rib of a flanged beam.

d, overall depth of beam or thickness of slab.

d_n, distance from compressive edge to neutral plane $= n_1 d_1$.

d_s, thickness of flange of a flanged beam.

d_1, d_1', actual effective depth and effective depth required respectively.

d_2, distance of compressive reinforcement below top of beam or of tensile reinforcement above bottom of beam.

F, coefficient in formula for A_{st} in rectangular beams with compressive reinforcement $= \frac{4b}{b_r}\gamma$.

K, coefficient relating to dimensions of flange of a flanged beam.

l_a, lever arm of compressive resistance of concrete $= a_1 d_1$.

M, applied bending moment; M', applied bending moment per unit width.

M_r, moment of resistance; M_r', moment of resistance of rectangular beam of unit width.

m, modular ratio $(= 15)$.

n_1, neutral-plane factor $= d_n/d_1$.

p_{cb}, permissible compressive stress in concrete in bending.

p_{sc}, p_{st}, permissible or actual compressive and tensile stresses respectively in reinforcement. $p_{sc}' = p_{sc}\frac{m-1}{m}$.

Q, shearing force; q, shearing stress.

Q_1, bending-moment factor $= \frac{M}{bd_1^2}$ or $\frac{M}{b_r d_1^2}$.

Q_c, moment-of-resistance factor (compression); Q', moment-of-resistance factor for rectangular beam of unit width.

q, shearing stress.

r_L, limiting proportion of tensile reinforcement for balanced design $= \frac{A_{st}}{bd_1}$ or $\frac{A_{st}}{b_r d_1}$.

* The derivation of the load-factor formulæ are given in "Explanatory Handbook on the B.S. Code of Practice," by Scott, Glanville and Thomas.

r_1, r_2, proportion of tensile and compressive reinforcement $= \frac{A_{st}}{bd_1}$ and $\frac{A_{sc}}{bd_1}$ respectively.

s', ratio of thickness of flange to effective depth $= \frac{d_s}{d_1}$.

γ, factor in B.S. Code relating to dimensions of flanged beams.

In the basic formulæ any mutually-consistent units may be used, but in formulæ containing numerical terms the units must be inches and pounds.

Rectangular Beams.

Moment of Resistance.—The moment of resistance M_r of a rectangular beam reinforced only in tension as expressed by formula (1) of the Code, which applies if it is based on the tensile resistance, is $A_{st}p_{st}l_a$, in which l_a is the lever arm and is $d_1 - \frac{3A_{st}p_{st}}{4bp_{cb}}$. If the strength of the beam is based on its compressive resistance, the moment of resistance as given by formula (2) of the Code is $\frac{p_{cb}bd_1^2}{4}$. If a moment-of-resistance factor $Q_c = 0{\cdot}25p_{cb}$ is substituted in the foregoing expressions, the resulting basic formulæ are

$$M_r = A_{st}p_{st}\left(d_1 - \frac{3A_{st}p_{st}}{16Q_cb}\right) \quad . \quad . \quad . \quad . \quad (A1)$$

and

$$M_r = Q_cbd_1^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (A2)$$

The safe moment of resistance is the smaller of the bending moments given by (A1) and (A2). For a balanced design these moments are identical, that is the tensile and compressive resistances are equal if calculated with the permissible stresses. By equating (A1) and (A2) the corresponding value of A_{st} is $\frac{p_{cb}}{3p_{st}}bd_1$, and if this amount is expressed as a ratio of the effective area bd_1 of the cross section of the beam the limiting proportion $r_L\left(= \frac{A_{st}}{bd_1}\right)$ is $\frac{p_{cb}}{3p_{st}}$. Therefore, if the proportion of reinforcement in a beam is less than r_L, the tensile resistance of the reinforcement controls the strength and M_r is calculated from (A1); if it is greater than r_L the compressive resistance of the concrete controls and (A2) applies.

If a rectangular beam contains compressive reinforcement the moment of resistance as given by formula (3) of the Code is $0{\cdot}25p_{cb}bd_1^2 + A_{sc}p_{sc}(d_1 - d_2)$, which can be rewritten in the form

$$M_r = Q_cbd_1^2 + A_{sc}p_{cs}(d_1 - d_2) \quad . \quad . \quad . \quad . \quad (A3)$$

This applies so long as there is sufficient tensile reinforcement to produce at least an equal resistance. The minimum amount of A_{st} for this purpose is given by

$$A_{st} = r_Lbd_1 + A_{sc}\frac{p_{sc}}{p_{st}} \quad . \quad . \quad . \quad . \quad (A4)$$

in which the first term is the amount of tensile reinforcement required to balance the resistance of the concrete and the second term is the amount required to balance the resistance of the compressive reinforcement.

The compressive stress p_{sc} in the compressive reinforcement is in general limited to 18,000 lb. per square inch in mild steel bars not greater than $1\frac{1}{2}$ in. diameter, 16,000 lb. per square inch in larger mild steel bars, and 23,000 lb. per square inch in high-yield-stress bars, but the Code recommends that this stress shall also be not greater than $50{,}000\left(1 - \frac{d_2}{d_n}\right)$. If the depth to the neutral plane is assumed to be $0{\cdot}5d_1$, which is the limiting depth of the compressive zone recommended in the Code, the limiting value of the compressive stress is given by

$$p_{sc} = 50{,}000\left(1 - 2\frac{d_2}{d_1}\right) \quad . \quad . \quad . \quad . \quad (A5)$$

The limiting stress does not apply to ordinary designs, since the cover-ratio d_2/d_1 must exceed 0·32 (an uncommonly high ratio) for p_{sc} to be less than the stress of 18,000 lb. per square inch permitted in mild steel bars in compression. Likewise the cover-ratio for high-yield-stress bars must exceed 0·17 (also a high ratio) for the permissible stress given by formula (A5) to be less than 23,000 lb. per square inch.

The basic formulæ (A1) to (A4) are summarised in series (A) in *Table* I; values of the moment-of-resistance factor Q_c for various stresses p_{cb} and the limiting ratio r_L for various combinations of stresses p_{cb} and p_{st} are given in *Table* II.

The Code recommends that the area of compressive reinforcement shall not exceed 4 per cent. of the area *bd*. With this limiting amount the moment of resistance given by (A3) becomes $Q_c bd_1^2 + 0{\cdot}04bdp_{sc}(d_1 - d_2)$. Substituting $Q_c = 0{\cdot}25p_{cb}$ and $d = d_1 + d_2$, this expression can be rewritten in the form

$$M_r = \left\{\frac{p_{cb}}{4} + 0{\cdot}04p_{sc}\left[1 - \left(\frac{d_2}{d_1}\right)^2\right]\right\}bd_1^2 \quad . \quad . \quad . \quad (A6)$$

Similarly the area of tensile reinforcement given by formula (A4) becomes $r_L bd_1 + 0{\cdot}04bd\frac{p_{sc}}{p_{st}}$, which can be rewritten in the form

$$A_{st} = \left[\frac{p_{cb}}{3} + 0{\cdot}04\left(1 + \frac{d_2}{d_1}\right)\right]\frac{bd_1}{p_{st}} \quad . \quad . \quad . \quad (A7)$$

and

$$A_{sc} = 0{\cdot}04\left(1 + \frac{d_2}{d_1}\right)bd_1 \quad . \quad . \quad . \quad . \quad . \quad (A8)$$

Design Procedure.—The series of basic formulæ (A) for calculating the moment of resistance can be transposed and simplified to forms which are convenient for determining the size and amount of reinforcement required for a beam to resist an applied bending moment *M* in.-lb. The required effective depth d_1' is given by

$$d_1' = \sqrt{\frac{M}{Q_c b}} \quad . \quad . \quad . \quad . \quad . \quad (A9)$$

TABLE I.

RECTANGULAR BEAMS & SOLID SLABS — FORMULÆ FOR LOAD-FACTOR METHOD.

		BASIC FORMULÆ	ORDINARY GRADE 1 : 2 : 4 CONCRETE P_{cb} = 1000 LB. PER SQ. IN.			
d, A_{st}, A_{sc}, d_2, b, d_1		P_{st} = TENSILE STRESS IN BARS OR WIRES P_{sc} = COMPRESSIVE STRESS IN BARS P_{cb} = COMPRESSIVE STRESS IN CONCRETE	RECTANGULAR BEAMS: MILD STEEL BARS P_{st} = 20,000 LB./SQ.IN. P_{sc} = 18,000 " " "	RECTANGULAR BEAMS: HIGH-YIELD-STRESS BARS P_{st} = 30,000 LB./SQ.IN. P_{sc} = 23,000 " " "	SOLID SLABS A_{st} PER FOOT, d_1, b = 12 IN.: MILD STEEL BARS P_{st} = 20,000 LB./SQ.IN.	SOLID SLABS: HIGH YIELD STRESS BARS P_{st} = 30,000 LB./SQ.IN.
		SERIES A	SERIES B	SERIES C	SERIES D	SERIES E
MOMENT OF RESISTANCE M_r — TENSILE REINF'T. ONLY, $r_L = \frac{P_{cb}}{3P_{st}}$	$\frac{A_{st}}{bd_1} < r_L$	$M_r = A_{st}P_{st}\left(d_1 - \frac{3A_{st}P_{st}}{16Q_cb}\right)$	$r_L = 1/60$ $20{,}000A_{st}\left(d_1 - \frac{15A_{st}}{b}\right)$	$r_L = 1/90$ $30{,}000A_{st}\left(d_1 - \frac{22 \cdot 5A_{st}}{b}\right)$	$20{,}000A_{st}(d_1 - 1 \cdot 25A_{st})$	$30{,}000A_{st}(d_1 - 1 \cdot 875A_{st})$
	$\not< r_L$	$M_r = Q_c bd_1^2 \quad Q_c = \frac{P_{cb}}{4}$	$250\,bd_1^2$	$250\,bd_1^2$	$3000\,d_1^2$	$3000\,d_1^2$
WITH COMPRESSION REINFORCEMENT		$M_r = Q_c bd_1^2 + A_{sc}P_{sc}(d_1 - d_2)$ $\left[A_{st} \not< r_L bd_1 + \frac{P_{sc}}{P_{st}}A_{sc}\right]$	$250\left[bd_1^2 + 72A_{sc}(d_1 - d_2)\right]$ $\left[A_{st} \not< \frac{bd_1}{60} + 0 \cdot 9A_{sc}\right]$	$250\left[bd_1^2 + 92A_{sc}(d_1 - d_2)\right]$ $\left[A_{st} \not< \frac{bd_1}{90} + 0 \cdot 77A_{sc}\right]$	—	—
DESIGN FORMULÆ — EFFECTIVE DEPTH REQUIRED	d_1'	$\sqrt{\frac{M}{Q_c b}}$	$\sqrt{\frac{M}{250b}}$	$\sqrt{\frac{M}{250b}}$	$\sqrt{\frac{M}{3000}}$	$\sqrt{\frac{M}{3000}}$
EFFECTIVE DEPTH PROVIDED $d_1 < d_1'$	A_{sc}	$\frac{Q_1 - Q_c}{P_{sc}\left(1 - \frac{d_2}{d_1}\right)}bd_1 \quad Q_1 = \frac{M}{bd_1^2}$	$\frac{Q_1 - 250}{18{,}000(1 - d_2/d_1)}bd_1$	$\frac{Q_1 - 250}{23{,}000(1 - d_2/d_1)}bd_1$	—	—
		$[A_{sc}$ NOT TO EXCEED 4 PER CENT. OF $b(d_1 + d_2)]$				
	A_{st}	$r_L bd_1 + \frac{A_{sc}P_{sc}}{P_{st}}$	$\frac{bd_1}{60} + 0 \cdot 9A_{sc}$	$\frac{bd_1}{90} + 0 \cdot 77A_{sc}$		
$d_1 = d_1'$ $(A_{sc} = 0)$	A_{st}	$r_L bd_1$	$\frac{bd_1}{60}$	$\frac{bd_1}{90}$	$\frac{d_1}{5}$	$\frac{d_1}{7 \cdot 5}$
$d_1 > d_1'$ $(A_{sc} = 0)$	A_{st}	$F r_L bd_1$	$\frac{bd_1}{60}F$	$\frac{bd_1}{90}F$	$\frac{d_1}{5}F$	$\frac{d_1}{7 \cdot 5}F$
		$F = 2\left(1 - \sqrt{1 - \frac{3Q_1}{P_{cb}}}\right)$	$F = 2\left(1 - \sqrt{1 - \frac{Q_1}{[illegible]}}\right)$			

If the actual effective depth d_1 provided is not less than d_1', tensile reinforcement only is required. If $d_1 = d_1'$, a balanced design is obtained if the amount of tensile reinforcement is calculated from

$$A_{st} = \frac{p_{cb}}{3p_{st}} bd_1 = r_L bd_1 \quad . \quad . \quad . \quad . \quad (\mathrm{A}10)$$

If d_1 is greater than d_1', manipulation of (A1) gives

$$A_{st} = Fr_L bd_1 \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{A}11)$$

in which $F = 2\left(1 - \sqrt{1 - \frac{3Q_1}{p_{cb}}}\right)$ if $Q_1 = \frac{M}{bd_1^2}$.

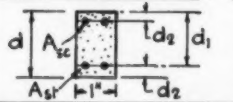
Compressive reinforcement is required if d_1 is less than d_1', and the amounts

TABLE II.

RECTANGULAR BEAMS — COEFFICIENTS FOR LOAD-FACTOR METHOD.

PERMISSIBLE COMPRESSIVE STRESS p_{cb} LB./SQ.IN.	M_r FACTOR Q_c LB/SQ.IN.	LIMITING PROPORTION OF TENSILE REINFORCEMENT $r_L = p_{cb}/3p_{st}$ — PERMISSIBLE TENSILE STRESS p_{st} LB. PER SQ.IN. 16,000	18,000	20,000	25,000	27,000	30,000	BENDING MOMENT FACTOR $Q_1 = \frac{M}{bd_1^2}$ LB./SQ.IN.	FACTOR $F = 2\left(1-\sqrt{1-\frac{3Q_1}{p_{cb}}}\right)$ — PERMISSIBLE COMPRESSIVE STRESS p_{cb} LB. PER SQ.IN. 750	1000	1250
750	188	0·016	0·014	0·013	0·010	0·009	0·008	100	0·45	0·33	0·26
800	200	17	15	13	11	10	09	110	0·50	0·36	0·28
850	213	18	16	14	11	11	10	120	0·56	0·40	0·31
900	225	19	17	15	12	11	10	130	0·61	0·44	0·34
950	238	20	18	16	13	12	11	140	0·67	0·48	0·37
								150	0·73	0·52	0·40
1000	250	0·021	0·019	0·017	0·013	0·012	0·011	160	0·80	0·56	0·43
1050	263	22	19	18	14	13	12	170	0·87	0·60	0·46
1100	275	23	20	18	15	14	12	180	0·94	0·64	0·49
1150	288	24	21	19	15	14	13	188	1·00	0·68	0·52
1200	300	25	22	20	16	15	13				
								200	COMPRESSION REINFORCEMENT REQUIRED IF $Q_1 > 188$	0·73	0·56
1250	313	0·026	0·023	0·021	0·017	0·015	0·014	210		0·78	0·59
1300	325	27	24	22	17	16	14	220		0·83	0·63
1350	338	28	25	23	18	17	15	230		0·88	0·66
1400	350	29	26	23	19	17	16	240		0·94	0·70
1450	368	30	27	24	19	18	16	250		1·00	0·74
1500	375	0·031	0·028	0·025	0·020	0·019	0·017	260		COMPRESSION REINFORCEMENT REQUIRED IF $Q_1 > 250$	0·77
1550	388	32	29	26	21	19	17	270			0·81
1600	400	33	30	27	21	20	18	280			0·86
1650	413	34	31	28	22	20	18	290			0·90
1700	425	35	31	28	23	21	19	300			0·95
1750	438	37	32	29	23	22	19	313			1·00

Table III.



RECTANGULAR BEAMS				MOMENT OF RESISTANCE AND REINFORCEMENT								LOAD-FACTOR METHOD.			
MILD STEEL BARS				p_{st} = 20,000 LB. PER SQ. IN.							p_{sc} = 18,000 LB. PER SQ. IN.				
				LOWER GRADE 1:2:4 CONCRETE p_{cb} = 750 LB. PER SQ. IN.				ORDINARY GRADE 1:2:4 CONCRETE p_{cb} = 1000 LB. PER SQ. IN.				HIGHER GRADE 1:2:4 CONCRETE p_{cb} = 1250 LB. PER SQ. IN.				COMPRESSION REINFORCEMENT [MAXIMUM AMOUNT FOR ALL CASES]
				TENSILE ONLY		TENS. & COMP.		TENSILE ONLY		TENS. & COMP.		TENSILE ONLY		TENS. & COMP.		
OVERALL DEPTH	d_2 (ASSUMED)		EFFECTIVE DEPTH (APPROX)	M_r'	A_{st}'	M_r'	A_{st}'	M_r'	A_{st}'	M_r'	A_{st}'	M_r'	A_{st}'	M_r'	A_{st}'	A_{sc}'
d IN.	COVER	BAR DIAM.	d_1 IN.	$187{\cdot}5d_1^2$ IN.-LB.	$d_1/80$ SQ. IN.	$Q'd_1^2$ IN.-LB.	$\frac{d_1}{80}+\frac{3{\cdot}6d}{100}$ SQ. IN.	$250d_1^2$ IN.-LB.	$d_1/60$ SQ. IN.	$Q'd_1^2$ IN.-LB.	$\frac{d_1}{60}+\frac{3{\cdot}6d}{100}$ SQ. IN.	$312{\cdot}5d_1^2$ IN.-LB.	$d_1/48$ SQ. IN.	$Q'd_1^2$ IN.-LB.	$\frac{d_1}{48}+\frac{3{\cdot}6d}{100}$ SQ. IN.	$=0{\cdot}04d$ SQ. IN.
12			10·5	20,700	0·131	98,900	0·563	27,600	0·175	106,000	0·607	34,500	0·219	112,000	0·651	0·48
14	1 IN. (MIN.)	≯ 1 IN.	12·5	29,200	0·156	140,000	0·661	39,000	0·208	150,000	0·713	48,800	0·260	160,000	0·765	0·56
15			13·5	34,100	0·169	163,000	0·709	45,500	0·225	175,000	0·765	57,000	0·281	186,000	0·821	0·60
16	$d_2 = 1\frac{1}{2}$ IN.		14·5	39,400	0·181	189,000	0·757	52,500	0·242	202,000	0·818	65,700	0·302	215,000	0·878	0·64
18			16·5	51,000	0·206	245,000	0·853	68,000	0·275	262,500	0·922	85,000	0·344	273,000	0·991	0·72
20			18·13	61,600	0·227	297,000	0·947	81,300	0·302	316,000	1·022	103,000	0·378	338,000	1·098	0·80
21			19·13	68,600	0·238	330,000	0·994	91,500	0·319	352,000	1·075	114,000	0·399	376,000	1·155	0·84
22	$1\frac{1}{4}$ IN.	≯ $1\frac{1}{4}$ IN.	20·13	75,900	0·252	365,000	1·045	101,000	0·336	390,000	1·129	126,000	0·420	416,000	1·213	0·88
24			22·13	91,800	0·277	442,000	1·132	122,000	0·369	467,000	1·234	153,000	0·461	503,000	1·326	0·96
26	$d_2 = 1\frac{7}{8}$ IN.		24·13	109,000	0·302	525,000	1·237	145,000	0·402	562,000	1·337	182,000	0·503	598,000	1·438	1·04
27			25·13	118,000	0·314	571,000	1·285	158,000	0·419	611,000	1·390	198,000	0·523	650,000	1·494	1·08
28			26·13	128,000	0·327	614,000	1·347	170,000	0·436	658,000	1·456	213,000	0·545	698,000	1·565	1·12
30			27·75	144,000	0·347	695,000	1·417	192,000	0·463	743,000	1·533	241,000	0·578	791,000	1·448	1·20
32	$1\frac{1}{2}$ IN.	≯ $1\frac{1}{2}$ IN.	29·75	165,000	0·372	795,000	1·522	220,000	0·496	850,000	1·646	276,000	0·620	905,000	1·770	1·28
33			30·75	177,000	0·384	852,000	1·572	236,000	0·513	911,000	1·701	295,000	0·640	970,000	1·820	1·32
34	$d_2 = 2\frac{1}{4}$ IN.		31·75	189,000	0·397	910,000	1·622	252,000	0·529	975,000	1·754	315,000	0·661	1,040,000	1·886	1·36
36			33·75	214,000	0·422	1,060,000	1·717	285,000	0·563	1,110,000	1·857	356,000	0·703	1,182,000	1·998	1·44

M_r' = MOMENT OF RESISTANCE PER INCH WIDTH. $Q' = 0{\cdot}25\,p_{cb} + 720\left[1-\left(\frac{d_s}{d_1}\right)^2\right]$

A_{st}' = AREA OF TENSILE REINFORCEMENT PER INCH WIDTH. A_{sc}' = AREA OF COMPRESSION REINFORCEMENT PER INCH WIDTH } MINIMUM REQUIRED TO PRODUCE TABULATED M_r'

of tensile and compressive reinforcement are given by

$$A_{sc} = \frac{Q_1 - Q_c}{p_{sc}\left(1 - \frac{d_2}{d_1}\right)} bd_1 \quad . \quad . \quad . \quad . \quad (A12)$$

$$A_{st} = r_L bd_1 + \frac{p_{sc}}{p_{st}} A_{sc} \quad . \quad . \quad . \quad . \quad (A13)$$

The basic formulæ (A9) to (A13) are summarised in series (A) in *Table* I.

MILD STEEL BARS AND 1 : 2 : 4 CONCRETE.—The basic formulæ (A1) to (A3) and (A9) to (A13) can be further simplified for common conditions by substituting numerical values for the stresses and factors based on the stresses. If the beam is of ordinary-grade 1 : 2 : 4 concrete, the permissible compressive stress p_{cb} in bending is 1000 lb. per square inch, if the effects of wind are excluded from the calculation of the applied bending moment M. If the reinforcement consists of mild steel bars not greater than $1\frac{1}{2}$ in. diameter, the permissible tensile stress p_{st} is 20,000 lb. per square inch, and the permissible compressive stress p_{sc} [if formula (A5) does not apply] is 18,000 lb. per square inch. (If the effects of wind are included all the foregoing stressss may be increased by 25 per cent.) For these stresses $Q_c = 250$ lb. per square inch, $r_L = \frac{1}{60}$, and $F = 2\left(1 - \sqrt{1 - \frac{Q_1}{333}}\right)$. Substitution of these numerical values in the basic formulæ in series (A) in *Table* I gives the design formulæ in series (B) in the table. Values of the coefficient F for various values of Q_1 and for p_{cb} = 750 lb., 1000 lb., and 1250 lb. per square inch are given in *Table* II.

The moment of resistance M_r' and areas of reinforcement A_{st}' and A_{sc}' for rectangular beams of various depths and unit width ($b = 1$ in.) reinforced with mild steel bars not greater than $1\frac{1}{2}$ in. diameter are given in *Table* III for 1 : 2 : 4 concrete in which the permissible compressive stresses are 1000 lb. per square inch (ordinary grade), 750 lb. per square inch (lower grade), and 1250 lb. per square inch (higher grade). For beams with tensile reinforcement only, M_r' is $\frac{p_{cb}}{4}d_1^2$ and A_{st}' is $\frac{p_{cb}d_1}{60,000}$. For beams with the greatest amount of compressive reinforcement, namely, $A_{sc}' = 0{\cdot}04\left(1 + \frac{d_2}{d_1}\right)d_1$, substitution in (A6) and (A7) gives

$$M_r' = \left\{\frac{p_{cb}}{4} + 720\left[1 - \left(\frac{d_2}{d_1}\right)^2\right]\right\}d_1^2 \text{ and } A_{st}' = \left[\frac{p_{cb}}{3} + 0{\cdot}04\left(1 + \frac{d_2}{d_1}\right)\right]\frac{d_1}{20,000}.$$

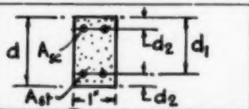
The data in *Table* III relating to beams with mild steel reinforcement are calculated from the foregoing expressions. *Table* III gives the moments of resistance M_r' and the corresponding areas of tensile reinforcement A_{st}' for rectangular beams of unit width ($b = 1$ in.) at various common stresses without compressive reinforcement, and with the limiting amount of compressive reinforcement, namely, $A_{sc}' = 0{\cdot}04b$. The values in *Table* III must be multiplied by the width of the beam to give the actual moment of resistance and the corresponding areas of tensile and compressive reinforcement.

It is assumed in the table that the distance frcm the tensile edge of the

TABLE IV.

RECTANGULAR BEAMS. MOMENT OF RESISTANCE AND REINFORCEMENT — LOAD-FACTOR METHOD.

HIGH-YIELD-STRESS BARS. p_{st} = 30,000 LB. PER SQ. IN. p_{sc} = 23,000 LB. PER SQ. IN.



Overall depth d in.	d_2 (assumed) cover	d_2 (assumed) bar diam.	Effective depth (approx.) d_1 in.	Lower grade 1:2:4 concrete p_{cb} = 750 lb. per sq. in. — Tensile only M_r' $187{\cdot}5d_1^2$ in.-lb.	Tensile only A_{st}' $d_1/120$ sq. in.	Tens. & comp. M_r' $Q'd_1^2$ in.-lb.	Tens. & comp. A_{st}' $\frac{d_1}{120}+\frac{3{\cdot}07d}{100}$ sq. in.	Ordinary grade 1:2:4 concrete p_{cb} = 1000 lb. per sq. in. — Tensile only M_r' $250d_1^2$ in.-lb.	Tensile only A_{st}' $d_1/90$ sq. in.	Tens. & comp. M_r' $Q'd_1^2$ in.-lb.	Tens. & comp. A_{st}' $\frac{d_1}{90}+\frac{3{\cdot}07d}{100}$ sq. in.	Higher grade 1:2:4 concrete p_{cb} = 1250 lb. per sq. in. — Tensile only M_r' $312{\cdot}5d_1^2$ in.-lb.	Tensile only A_{st}' $d_1/72$ sq. in.	Tens. & comp. M_r' $Q'd_1^2$ in.-lb.	Tens. & comp. A_{st}' $\frac{d_1}{72}+\frac{3{\cdot}07d}{100}$ sq. in.	Compression reinforcement [Maximum amount for all cases] A_{sc}' = 0·04d sq. in.
12	1 in. (min.)	$\frac{3}{4}$ in.	10·63	21,200	0·089	124,000	0·457	28,200	0·118	130,500	0·486	35,400	0·148	137,500	0·516	0·48
14			12·63	30,000	0·105	175,500	0·535	40,000	0·140	185,000	0·570	50,000	0·175	196,000	0·605	0·56
15			13·63	34,900	0·114	214,000	0·575	46,500	0·151	216,000	0·612	58,100	0·190	228,000	0·651	0·60
16	$d_2 = 1\frac{3}{8}$ in.		14·63	40,300	0·122	236,500	0·612	53,800	0·163	250,000	0·653	67,200	0·203	264,000	0·693	0·64
18			16·63	52,000	0·139	306,000	0·691	69,200	0·185	322,000	0·737	86,500	0·232	340,000	0·784	0·72
20	1 in.	1 in.	18·50	64,100	0·154	377,000	0·767	85,500	0·206	398,000	0·819	108,000	0·257	418,000	0·870	0·80
21			19·50	71,200	0·163	419,000	0·808	95,000	0·217	443,000	0·862	119,000	0·271	466,000	0·916	0·84
22			20·50	79,600	0·172	463,000	0·847	105,000	0·228	489,000	0·903	131,000	0·285	515,000	0·960	0·88
24			22·50	95,000	0·188	559,000	0·823	126,700	0·250	590,000	0·985	158,000	0·312	622,000	1·047	0·96
26	$d_2 = 1\frac{1}{2}$ in.		24·50	112,000	0·204	663,000	1·002	150,000	0·272	700,000	1·070	187,000	0·340	737,000	1·138	1·04
27			25·50	122,000	0·213	718,000	1·042	162,000	0·283	757,000	1·112	207,000	0·354	798,000	1·183	1·08
28			26·50	131,000	0·221	774,000	1·080	175,000	0·294	815,000	1·153	219,000	0·368	860,000	1·227	1·12
30	$1\frac{1}{4}$ in.	$1\frac{1}{4}$ in.	28·13	148,000	0·234	872,000	1·154	197,000	0·312	920,000	1·232	248,000	0·391	970,000	1·311	1·20
32			30·13	171,000	0·251	1,005,000	1·232	227,000	0·335	1,060,000	1·316	284,000	0·419	1,118,000	1·400	1·28
33			31·13	181,000	0·259	1,007,000	1·269	242,000	0·346	1,130000	1·356	303,000	0·433	1,190,000	1·443	1·32
34	$d_2 = 1\frac{7}{8}$ in.		32·13	194,000	0·268	1,145,000	1·310	259,000	0·357	1,210,000	1·399	323,000	0·447	1,275,000	1·489	1·36
36			34·13	222,000	0·284	1,305,000	1·387	270,000	0·379	1,375,000	1·482	370,000	0·475	1,450,000	1·578	1·44

M_r' = MOMENT OF RESISTANCE PER INCH WIDTH. $Q' = 0{\cdot}25p_{cb} + 920\left[1-\left(\frac{d_2}{d_1}\right)^2\right]$

A_{st}' = AREA OF TENSILE REINFORCEMENT PER INCH WIDTH. A_{sc}' = AREA OF COMPRESSION REINFORCEMENT PER INCH WIDTH. } MINIMUM REQUIRED TO PRODUCE TABULATED M_r'.

beam to the centre of the tensile reinforcement is equal to the distance d_2 from the compressive edge to the centre of the compressive reinforcement. This distance is the sum of the cover of concrete over the bars and half the diameter of the bars (assumed to be in one layer); a reasonable value for d_2 is assumed for each beam and usually a variation from this value of, say, $\pm\frac{1}{8}$ in. has no significant effect on the tabulated data; if d_2 is much greater than is assumed, the data should be adjusted to conform to the actual value.

HIGH-YIELD-STRESS BARS AND 1 : 2 : 4 CONCRETE.—For this condition p_{cb} is 1000 lb. per square inch for ordinary-grade concrete and, if the yield stress of the bars is not less than 60,000 lb. per square inch, p_{st} is 30,000 lb. and p_{sc} [assuming (A5) does not apply] is 23,000 lb. per square inch. These stresses apply to bars of any size and it is assumed, as before, that the effects of wind are excluded from the calculation of M. The numerical factors are $Q_c = 250$ lb. per square inch and $r_L = \frac{1}{90}$; F is as before. Substitution in the basic formulæ in series (A) in *Table* I gives the formulæ in series (C) in the table.

The data in *Table* IV relating to rectangular beams reinforced with bars having a yield stress of not less than 60,000 lb. per square inch are determined in a manner similar to that for mild steel bars but using the stresses suitable for high-yield-stress bars. For beams with tensile reinforcement only M_r' is $\frac{p_{cb}}{4}d_1^2$ and A_{st}' is $\frac{p_{cb}d_1}{90{,}000}$. For beams with compressive reinforcement A_{sc} equal to $0{\cdot}04\left(1 + \frac{d_2}{d_1}\right)d_1$, formulæ (A6) and (A7) give

$$M_r' = \left\{\frac{p_{cb}}{4} + 920\left[1 - \left(\frac{d_2}{d_1}\right)^2\right]\right\}d_1^2 \text{ and } A_{st}' = \left[\frac{p_{cb}}{3} + 0{\cdot}04\left(1 + \frac{d_2}{d_1}\right)\right]\frac{d_1}{30{,}000},$$

and these formulæ are used for the calculation of the data relating to beams with high-yield-stress bars in *Table* IV.

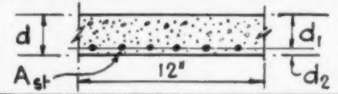
Solid Slabs.

Solid slabs can be designed as rectangular beams of 1 ft. width if the applied bending moment M is in inch-pounds per foot width of slab and A_{st} is the area of reinforcement in square inches per foot width. The basic formulæ in series (A) in *Table* I apply directly to solid slabs if 12 in. is substituted for b. Similarly the formulæ in series (D) in *Table* I for mild steel bars and in series (E) in *Table* I for high-yield-stress bars or wire are obtained from formulæ series (B) and (C) respectively by substituting $b = 12$ in. No formulæ are given for solid slabs with compressive reinforcement as such slabs are uncommon and uneconomical; any special case of such a slab can be designed directly from the basic formulæ.

In *Table* V are given the moments of resistance M_r and areas of tensile reinforcement A_{st} for solid slabs 1 ft. wide and of various thickness; M_r is calculated from the expression $3000d_1^2$ for 1 : 2 : 4 concrete of ordinary grade and from similar expressions for concrete of lower and higher grades. The areas of tensile reinforcement per foot of width are calculated from $\frac{p_{cb}d_1}{5000}$ for mild steel bars not greater than $1\frac{1}{2}$ in. diameter and from $\frac{p_{cb}d_1}{7500}$ for high-yield-stress bars.

TABLE V.

SOLID SLABS. MOMENT OF RESISTANCE AND REINFORCEMENT. LOAD-FACTOR METHOD.



THICKNESS	MILD STEEL BARS: COVER PLUS ½(BAR DIA.)	MILD STEEL BARS: EFFECTIVE DEPTH	HIGH-YIELD-STRESS BARS (OR WIRE): COVER PLUS ½(BAR DIA.)	HIGH-YIELD-STRESS BARS (OR WIRE): EFFECTIVE DEPTH	LOWER GRADE 1:2:4 CONCRETE p_{cb} = 750 LB. PER SQ. IN. — MILD STEEL BARS p_{st} = 20,000 LB. PER SQ. IN.		LOWER GRADE — HIGH-YIELD-STRESS BARS OR WIRE p_{st} = 30,000 LB. PER SQ. IN.		ORDINARY GRADE 1:2:4 CONCRETE p_{cb} = 1000 LB. PER SQ. IN. — MILD STEEL BARS p_{st} = 20,000 LB. PER SQ. IN.		ORDINARY GRADE — HIGH-YIELD-STRESS BARS OR WIRE p_{st} = 30,000 LB. PER SQ. IN.		HIGHER GRADE 1:2:4 CONCRETE p_{cb} = 1250 LB. PER SQ. IN. — MILD STEEL BARS p_{st} = 20,000 LB. PER SQ. IN.		HIGHER GRADE — HIGH-YIELD-STRESS BARS OR WIRE p_{st} = 30,000 LB. PER SQ. IN.	
d IN.	d_2	d_1 IN.	d_2	d_1 IN.	M_r $2250d_1^2$ IN.-LB.	A_{st} $0{\cdot}15d_1$ SQ. IN.	M_r $2250d_1^2$ IN.-LB.	A_{st} $0{\cdot}10d_1$ SQ. IN.	M_r $3000d_1^2$ IN.-LB.	A_{st} $0{\cdot}20d_1$ SQ. IN.	M_r $3000d_1^2$ IN.-LB.	A_{st} $0{\cdot}133d_1$ SQ. IN.	M_r $3750d_1^2$ IN.-LB.	A_{st} $0{\cdot}25d_1$ SQ. IN.	M_r $3750d_1^2$ IN.-LB.	A_{st} $0{\cdot}167d_1$ SQ. IN.
3	≮ ¾ IN.	2·25	≮ ¾ IN.	2·25	11,400	0·338	11,400	0·225	15,200	0·450	15,200	0·300	19,000	0·563	19,000	0·375
3½		2·75		2·75	17,000	0·413	17,000	0·275	22,600	0·550	22,600	0·367	28,400	0·688	28,400	0·485
4	≮ 1 IN.	3·00	≮ ¾ IN.	3·25	20,300	0·450	23,800	0·325	27,000	0·600	31,800	0·433	33,800	0·750	39,600	0·543
4½		3·50		3·75	27,600	0·525	31,600	0·375	36,800	0·700	42,200	0·500	45,900	0·875	52,600	0·625
5		4·00		4·25	36,000	0·600	40,600	0·425	48,000	0·800	54,000	0·567	60,000	1·000	67,500	0·708
5½		4·50		4·75	46,600	0·675	50,800	0·475	60,800	0·900	67,500	0·634	76,000	1·125	84,500	0·792
6	≮ 1⅛ IN.	4·88	≮ 1 IN.	5·00	54,800	0·731	56,400	0·500	71,600	0·975	75,000	0·667	89,500	1·220	93,600	0·833
7		5·88		6·00	78,100	0·881	81,100	0·600	104,100	1·175	108,000	0·800	130,000	1·470	135,000	1·000
8		6·88		7·00	106,800	1·031	110,500	0·700	142,200	1·375	147,000	0·935	177,500	1·720	184,000	1·167
9	≮ 1½ IN.	7·50	≮ 1⅛ IN.	7·88	126,300	1·125	139,500	0·788	168,600	1·500	186,000	1·050	210,000	1·875	232,000	1·313
10		8·50		8·88	162,600	1·275	177,500	0·888	216,600	1·700	236,400	1·185	270,000	2·125	295,000	1·479
11		9·50		9·88	204,000	1·425	220,500	0·988	271,500	1·900	293,400	1·320	339,000	2·375	366,000	1·646
12		10·50		10·88	248,000	1·575	266,000	1·088	331,500	2·100	354,000	1·450	413,000	2·625	442,000	1·813
14	≮ 1⅞ IN.	12·13	≮ 1½ IN.	12·50	332,000	1·819	351,500	1·250	442,500	2·425	468,000	1·667	553,000	3·033	585,000	2·083
15		13·13		13·50	380,000	1·969	410,000	1·350	519,000	2·625	546,000	1·800	649,000	3·283	682,000	2·250
16		14·13		14·50	450,000	2·119	474,000	1·450	600,000	2·825	630,000	1·930	750,000	3·533	786,000	2·417
18		16·13		16·50	588,000	2·419	613,000	1·650	783,000	3·225	816,000	2·200	978,000	4·033	1,020,000	2·750

COVER OF CONCRETE = DIAM. OF BAR (OR WIRE) AND ≮ ½ IN.

M_r = MOMENT OF RESISTANCE PER FOOT WIDTH. A_{st} = AREA OF TENSILE REINFORCEMENT PER FOOT WIDTH. (MINIMUM REQUIRED TO PROVIDE TABULATED M_r)

Flanged Beams.

Moment of Resistance.—Flanged beams include tee-beams, ell-beams, I-beams, inverted channels, and the like. The moments of resistance M_r of a flanged beam as given by formulæ (4) and (5) of the Code are $A_{st}p_{st}(d_1 - 0{\cdot}5d_s)$ if they are based on the tensile resistance of the reinforcement, and $\gamma p_{cb}bd_1^2$ if based on the compressive strength of the concrete, where the coefficient γ is $\frac{b_r}{4b} + \frac{1}{3}\left(1 - \frac{b_r}{b}\right)\left[2\frac{d_s}{d_1} - \left(\frac{d_s}{d_1}\right)^2\right]$ as given by formula (7) of the Code. If the beam contains compressive reinforcement the moment of resistance as given by formula (6) of the Code is $\gamma p_{cb}bd_1^2 + A_{sc}p_{sc}(d_1 - d_2)$. As in the case of rectangular beams, these expressions can be simplified to give formulæ which bear some similarity to the corresponding formulæ for rectangular beams.

If $4\gamma\frac{b}{b_r} = K$, $s' = \frac{d_s}{d_1}$, and $Q_c = \frac{p_{cb}}{4}$ (as before),

then
$$K = 1 - \frac{4s'}{3}\left(\frac{b}{b_r} - 1\right)(2 - s')$$
and therefore, with tensile reinforcement only,
$$M_r = KQ_cb_rd_1^2 \qquad \text{(F1)}$$

Formula (F1) applies when the compressive resistance of the concrete controls the design, as does formula (5) of the Code from which it is derived. If the tensile resistance of the reinforcement controls the design, formula (4) of the Code may be written in the form

$$M_r = A_{st}p_{st}d_1(1 - 0{\cdot}5s') \qquad \text{(F2)}$$

It can be shown that the limiting proportion of tensile reinforcement $\frac{A_{st}}{b_rd_1}$, which determines whether (F1) or (F2) applies, is given by

$$r_L = \frac{KQ_cd_1}{p_{st}(d_1 - 0{\cdot}5d_s)} = \frac{KQ_c}{p_{st}(1 - 0{\cdot}5s')} \qquad \text{(F3)}$$

The formula corresponding to formula (6) of the Code for a flanged beam with compressive reinforcement is

$$M_r = KQ_cb_rd_1^2 + A_{sc}p_{sc}(d_1 - d_2) \qquad \text{(F4)}$$

which applies if there is sufficient tensile reinforcement to produce at least this resistance; the minimum amount required for this purpose is given by

$$A_{st} = \frac{KQ_cb_rd_1^2}{p_{st}(d_1 - 0{\cdot}5d_s)} + \frac{A_{sc}p_{sc}}{p_{st}} \qquad \text{(F5)}$$

The basic formulæ (F1) to (F5) are summarised under series (F) in *Table* VI, which also gives values of K for various values of the ratio of b to b_r and of s'; intermediate values can be interpolated or can be calculated from formula (F1).

Design Procedure.—The dimensions of a flanged beam are generally determined from considerations other than the strength in bending; for example, resistance to shear may determine the size of the rib, and the thickness d_s of the slab may be determined by its span and loading. Therefore the dimensions d_1, d_s, b and b_r are known, or can be assumed, and design involves the determination of the amount of tensile reinforcement and the checking of the

TABLE VI.

FLANGED BEAMS. FORMULÆ FOR LOAD-FACTOR METHOD.

BASIC FORMULÆ

P_{st} = TENSILE STRESS IN BARS.

P_{sc} = COMPRESSIVE STRESS IN BARS $\not> 50{,}000\left(1-2\frac{d_2}{d_1}\right)$

P_{cb} = COMPRESSIVE STRESS IN CONCRETE.

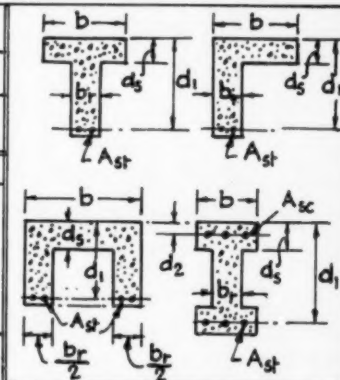
ORDINARY GRADE 1 : 2 : 4 CONCRETE P_{cb} = 1000 LB. PER SQ. IN.

				MILD STEEL BARS (DIAM. NOT GREATER THAN 1½ IN.)	HIGH-YIELD-STRESS BARS (ALL SIZES)
				P_{st} = 20,000 LB./SQ. IN. P_{sc} = 18,000 " " "	P_{st} = 30,000 LB./SQ. IN. P_{sc} = 23,000 " " "
		$\frac{A_{st}}{b_r d_1}$	SERIES F	SERIES G	SERIES H
MOMENT OF RESISTANCE M_r	TENSILE REINF'T. ONLY	$< r_L$	$M_r = A_{st} P_{st} d_1 (1 - s'/2)$	$20{,}000 A_{st}\left(d_1 - \frac{d_s}{2}\right)$	$30{,}000 A_{st}\left(d_1 - \frac{d_s}{2}\right)$
		$> r_L$	$M_r = K Q_c b_r d_1^2$	$250 K b_r d_1^2$	$250 K b_r d_1^2$
	WITH COMPRESSION REINFORCEMENT		$M_r = K Q_c b_r d_1^2 + A_{sc} P_{sc}(d_1 - d_2)$ $\left[A_{st} \not< r_L b_r d_1 + \frac{P_{sc}}{P_{st}} A_{sc}\right]$	$250 K b_r d_1^2 + 18{,}000 A_{sc}(d_1 - d_2)$ $\left[A_{st} \not< r_L b_r d_1 + 0{\cdot}9 A_{sc}\right]$	$250 K b_r d_1^2 + 23{,}000 A_{sc}(d_1 - d_2)$ $\left[A_{st} \not< r_L b_r d_1 + 0{\cdot}77 A_{sc}\right]$
DESIGN FORMULÆ	TENSILE REINF'T. ONLY	A_{st}	$\frac{M}{P_{st} d_1 (1 - s'/2)}$	$\frac{M}{20{,}000\left(d_1 - \frac{d_s}{2}\right)}$	$\frac{M}{30{,}000\left(d_1 - \frac{d_s}{2}\right)}$
	COMPRESSION REINF'T REQUIRED IF $K < \frac{Q_1}{Q_c}$	A_{st}	$r_L b_r d_1 + \frac{P_{sc}}{P_{st}} A_{sc}$	$r_L b_r d_1 + 0{\cdot}9 A_{sc}$	$r_L b_r d_1 + 0{\cdot}77 A_{sc}$
		A_{sc}	$\frac{(Q_1 - K Q_c) b_r d_1^2}{P_{sc}(d_1 - d_2)}$	$\frac{(Q_1 - 250 K) b_r d_1^2}{18{,}000(d_1 - d_2)}$	$\frac{(Q_1 - 250 K) b_r d_1^2}{23{,}000(d_1 - d_2)}$

$Q_c = \frac{P_{cb}}{4}$; $Q_1 = \frac{M}{b_r d_1^2}$; $\frac{d_s}{d_1} = s'$

$r_L = \frac{Q_c K}{P_{st}(1 - s'/2)}$; $K = 1 + \frac{4s'}{3}\left(\frac{b}{b_r} - 1\right)(2 - s')$

FORMULÆ IN SERIES (G) AND (H) INCORPORATING P_{sc} = 18,000 OR 23,000 LB. PER SQ. IN. APPLY ONLY IF $\frac{d_2}{d_1} \not> 0{\cdot}32$ FOR MILD STEEL BARS OR $\not> 0{\cdot}17$ FOR HIGH-YIELD-STRESS BARS.



VALUES OF K

$\frac{b}{b_r}$	$s' = d_s/d_1$ 0·1	0·2	0·3	0·4	0·5 OR MORE
1 (RECT.)	(1·0)	(1·0)	(1·0)	(1·0)	1·0
2	1·2	1·5	1·7	1·9	2·0
3	1·5	2·0	2·4	2·7	3·0
4	1·7	2·4	3·0	3·6	4·0
5	2·0	2·9	3·7	4·4	5·0
6	2·2	3·4	4·4	5·3	6·0
8	2·7	4·4	5·8	7·0	8·0
10	3·2	5·3	7·1	8·7	10·0
12	3·8	6·3	8·5	9·4	12·0
15	4·6	7·7	10·5	13·0	15·0

compressive resistance required for an applied bending moment of M in.-lb. Transposing formula (F2) gives the amount of tensile reinforcement, namely,

$$A_{st} = \frac{M}{p_{st}d_1(1 - 0{\cdot}5s')} \quad . \quad . \quad . \quad . \quad . \quad \text{(F6)}$$

The compressive resistance can be checked by comparing the value of K corresponding to the known values of the ratio of b to b_r and of s' [using formula (F1)] with the minimum value required, which is given by

$$K_{\text{min.}} = \frac{M}{Q_c b_r d_1^2} = \frac{Q_1}{Q_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(F7)}$$

in which $Q_1 = \dfrac{M}{b_r d_1^2}$.

If K is less than the value given by (F7), compressive reinforcement is required or the dimensions of the cross section of the beam must be increased. In tee-beams in ordinary floors it is rare for the resistance to bending to be limited by the compressive resistance; compressive reinforcement is not often required and is, in any case, uneconomical. Compressive reinforcement is more common in I-beams and other special beams and, if d_2 is less than $0{\cdot}5d_1$ the amount of compressive reinforcement required is given by

$$A_{sc} = \frac{Q_1 - KQ_c}{p_{sc}(d_1 - d_2)} b_r d_1^2 \quad . \quad . \quad . \quad . \quad . \quad \text{(F8)}$$

and the corresponding amount of tensile reinforcement is given by

$$A_{st} = r_L b_r d_1 + A_{sc}\frac{p_{sc}}{p_{st}} \quad . \quad . \quad . \quad . \quad . \quad \text{(F9)}$$

which is a modification of (F5). Formulæ (F6) to (F9) are summarised in series F, *Table* VI.

MILD STEEL BARS AND 1 : 2 : 4 CONCRETE.—As for rectangular beams in *Table* I, formulæ series (G) in *Table* VI are derived from the basic formulæ in series (F) by substituting $p_{cb} = 1000$ lb. per square inch (for 1 : 2 : 4 ordinary-grade concrete), $p_{st} = 20{,}000$ lb. and $p_{sc} = 18{,}000$ lb. per square inch for mild steel bars not more than 1½ in. diameter. The stress p_{sc} applies if the value of the ratio d_2/d_1 is not more than 0·32. Also $Q_c = 250$ lb. per square inch.

HIGH-YIELD-STRESS BARS AND 1 : 2 : 4 CONCRETE.—Similarly, formulæ series (H) in *Table* VI are derived from series (F) by substituting $p_{cb} = 1000$ lb. per square inch, $p_{st} = 30{,}000$ lb. and $p_{sc} = 23{,}000$ lb. per square inch for bars of all sizes having a yield stress of not less than 60,000 lb. per square inch. The stress p_{sc} applies if the value of the ratio d_2/d_1 is not more than 0·17. As before, $Q_c = 250$ lb. per square inch.

(To be continued.)

Soil-cement Piles.

CEMENT-stabilised soil has recently been used in the U.S.A. for the construction of piles. At first, the soil was removed from the pile-shaft by an auger and the hole filled with a mixture of cement and sand, but it has been found to be cheaper to combine the grout with the unexcavated soil, which is used as fine aggregate or filling. The equipment for forming piles consists of a truck-mounted rotating mixing-head driven by a hollow drill-shaft through which grout is forced.

The equipment available (*Fig.* 1) can reach a depth of 30 ft. in one operation, and this can be increased by using additional rods. The piles are formed by forcing the mixing-head into the soil and supplying grout to the head through the hollow shaft. Immediately the head is withdrawn, reinforcement may be placed in the pile.

The quantity of grout used varies between 30 per cent. of the volume of the pile in the case of fine sands and compacted clays and 60 per cent. in the case of loose sands and soft clays. The grout is mixed in the proportions of 2 cu. ft. of cement and 1 cu. ft. of siliceous material to $12\frac{1}{2}$ Imperial gallons of water; a colloidal chemical comprises 1 per cent. of the weight of the dry materials. Richer mixtures are required for soft clays and silts.

The depth to which the piles may be formed is limited by the torsional force available to operate the mixing-head. Depths of 58 ft. have been reached with a 12-in. head, 50 ft. with an 18-in. head, and 30 ft. with a 24-in. head.

The method has been used at Mobile, Alabama, to form sheet-pile walls around an area which was subsequently excavated, at Lake Erie to control beach erosion, and in Michigan to reduce the earth pressure on a bridge abutment. It has been developed in the U.S.A. by Intrusion-Prepakt, Inc., and Messrs. Edmund Nuttall, Sons & Co. (London), Ltd., have the rights for the process in Great Britain.

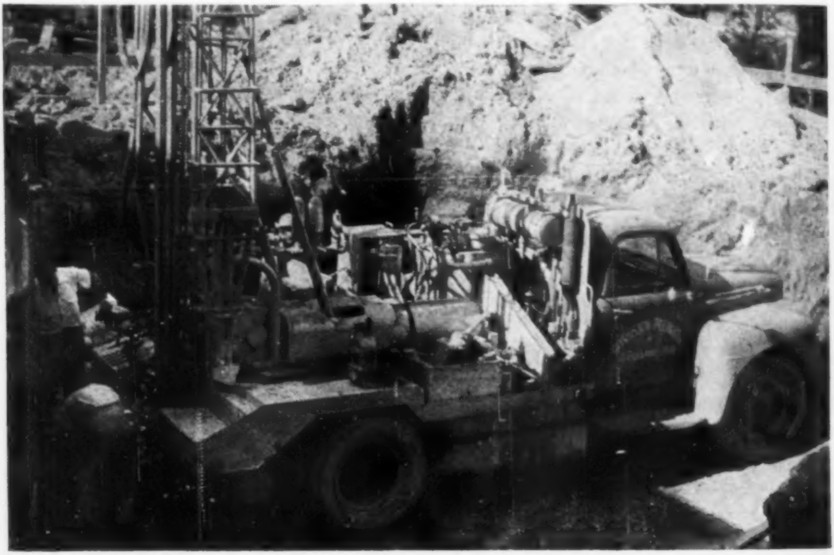


Fig. 1.

The Maidstone By-pass Road.

BRIDGES FOR THE EASTERN SECTION.

THE eastern section of the Maidstone by-pass road, work on which was recently started and is expected to be completed in about two years, will be about $5\frac{1}{4}$ miles long. The road will have an overall width of 88 ft. and will comprise two carriageways each 24 ft. wide, a central reservation, hard shoulders, marginal strips, and grass verges. There will be eight bridges, all of concrete construction, which will allow the road to pass over and under existing roads and over a railway.

The Ministry of Transport and Civil Aviation appointed Messrs. Scott & Wilson, Kirkpatrick & Partners consulting engineers for the design and construction of these bridges and Messrs. Ansell & Bailey consulting architects to collaborate with the consulting engineers. The following notes relating to the bridges have

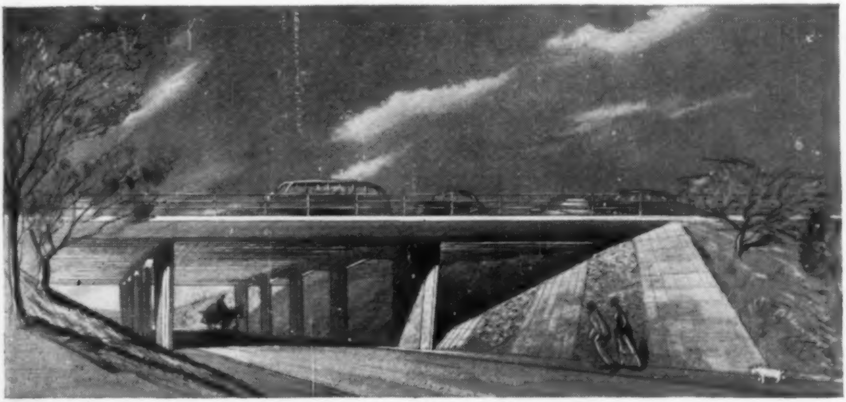


Fig. 1.—Beam-and-Slab Under-bridge.

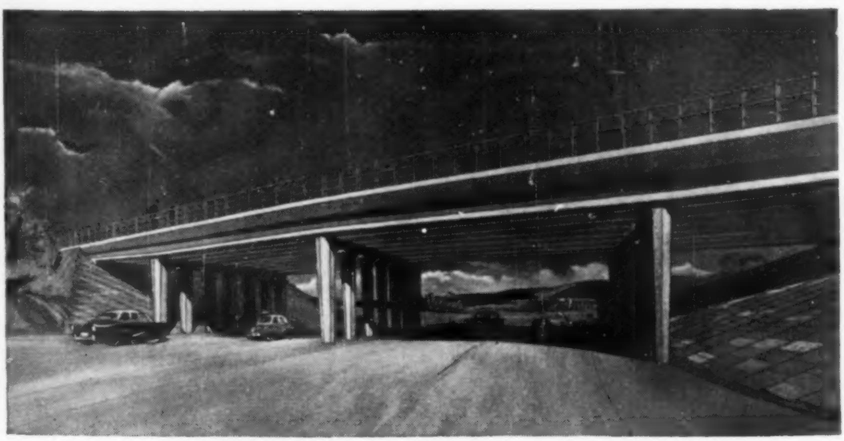


Fig. 2.—Beam-and-Slab Over-bridge.

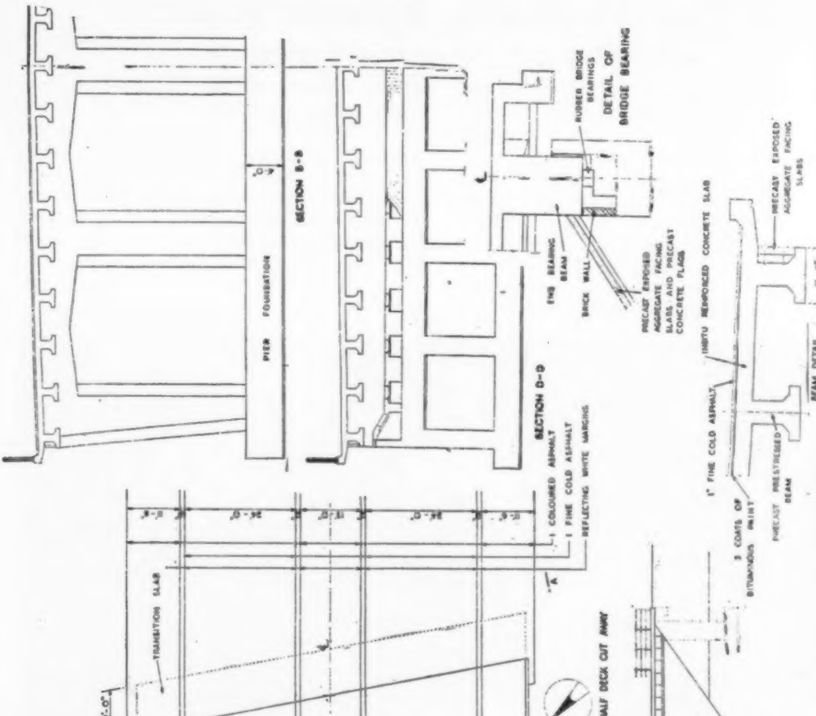


Fig. 3.—Typical Details of Bridges shown in Figs. 1 and 2.

Fig. 3.—Typical Details of Bridges shown in Figs. 1 and 2.

been supplied to us by the consulting engineers.

The consulting architects and consulting engineers have worked in close collaboration, with the object of designing structures which would be functional, quick to construct, and economical yet æsthetically pleasant.

The road passes through a beautiful part of the county of Kent and care has been taken to ensure that the facing materials will harmonise with the surrounding countryside. The sides of the bridges and the slopes of the embankments under the bridges are to be faced with exposed-aggregate panels and the piers are to be of white concrete with point-tooled surfaces. These colourful features will avoid the monotony of normal unrelieved dull grey concrete. Cantilevered slabs are provided at the edges of the decks wherever possible, together with welded-tube railings, to give an impression of lightness.

Because of large variations in the widths and angles of skew of the bridges, the ground conditions, and in some cases the limited headroom available, it was not structurally desirable or economically justifiable to standardise the designs as much as would have been preferred. Some standardisation was possible, however, and two basic types will be used.

Most of the bridges will be of the types shown in *Figs.* 1, 2, and 3. These will consist of precast prestressed beams which can be lifted into position, and a reinforced concrete deck-slab cast in place. The shuttering for the deck-slab will be supported by the beams, and no props will be required. This procedure is considered to be preferable to propping from the ground as, apart from the possibility of settlement and other considerations, it allows free access under the bridges during construction. Transverse bending moments will be distributed by the reinforced concrete deck-slab acting in conjunction with prestressed transverse stiffeners.

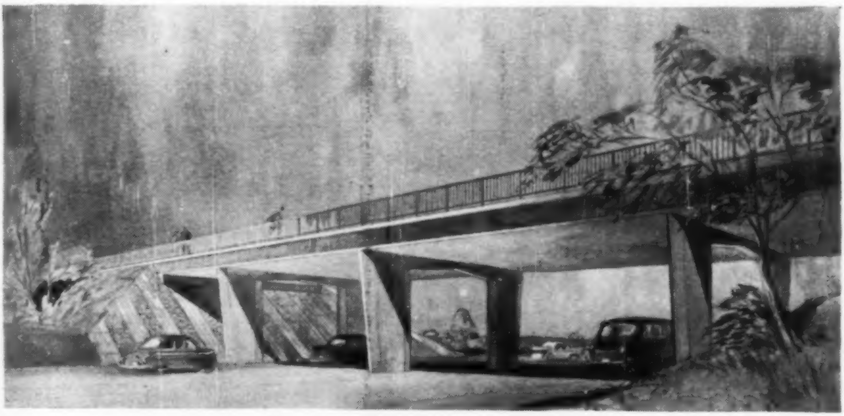


Fig. 4.—Hollow-slab Over-bridge.

The second type will consist of a continuous hollow reinforced concrete slab as shown in *Figs.* 4 and 5. The hollow slab will be formed by means of fibre tubes, which are considered to be structurally preferable to, and cheaper than, the conventional internal shuttering.

Chrismill bridge (*Figs.* 6 and 7), which will carry the road over a railway, is a notable exception to the standard types, neither of which is suitable. This bridge will have the large angle of skew of 70 deg. which, together with the limited structural depth, raised some interesting structural problems. The structure will consist of flat slabs, except for the part which crosses the railway tracks. This will consist of prestressed precast beams that can be placed in position without interfering with the railway traffic, and

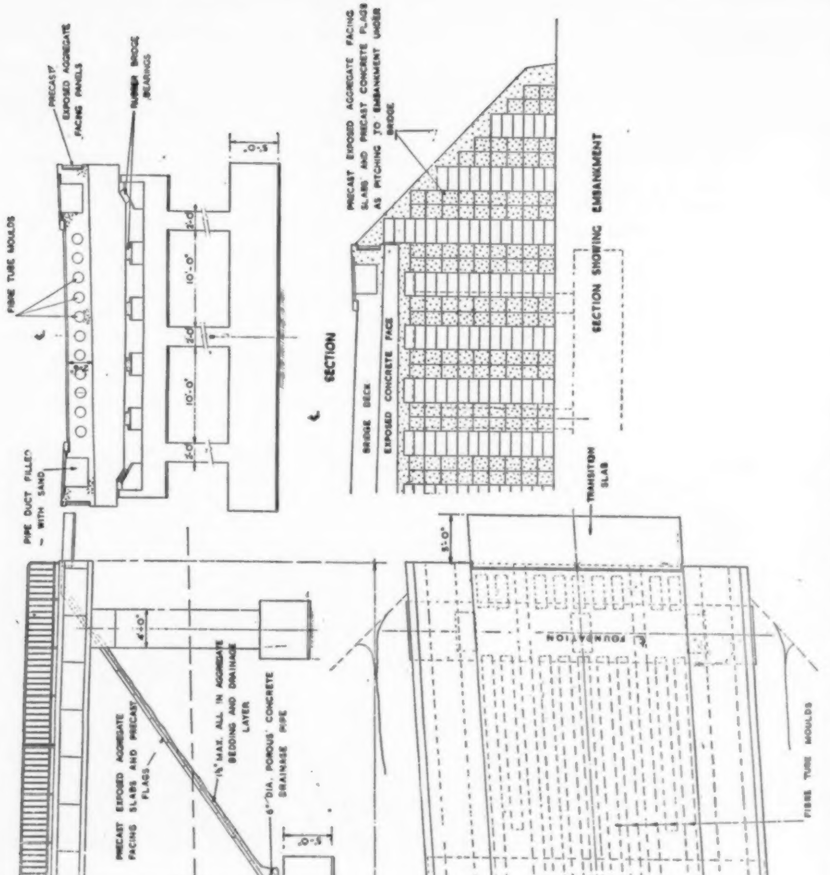
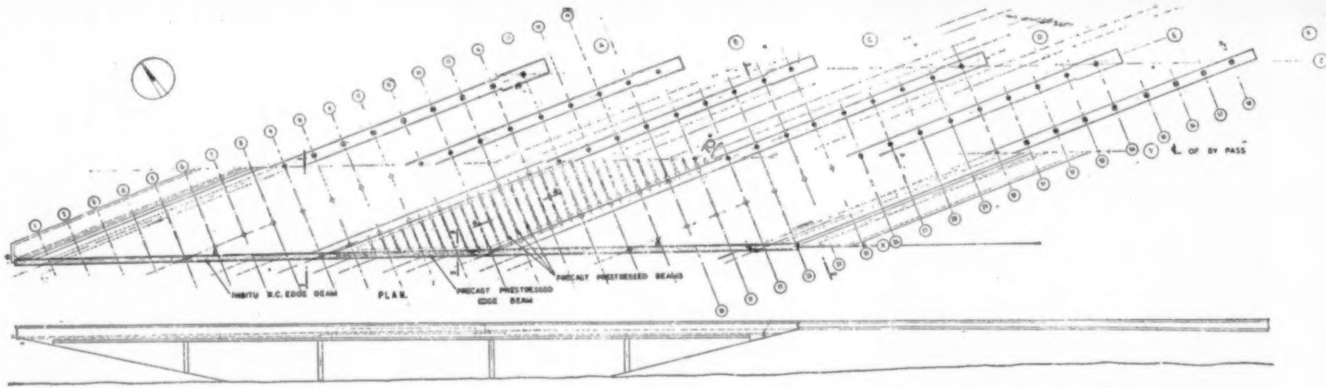
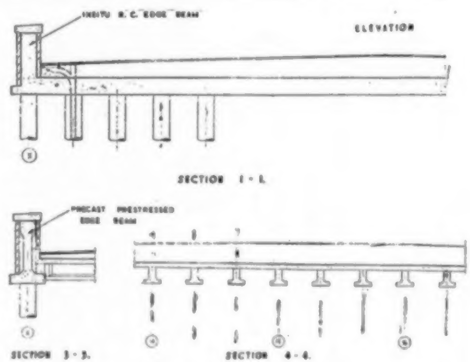


Fig. 5.—Typical Details of Type of Bridge shown in Fig. 4.

Fig. 5.—Typical Details of Type of Bridge shown in Fig. 4.





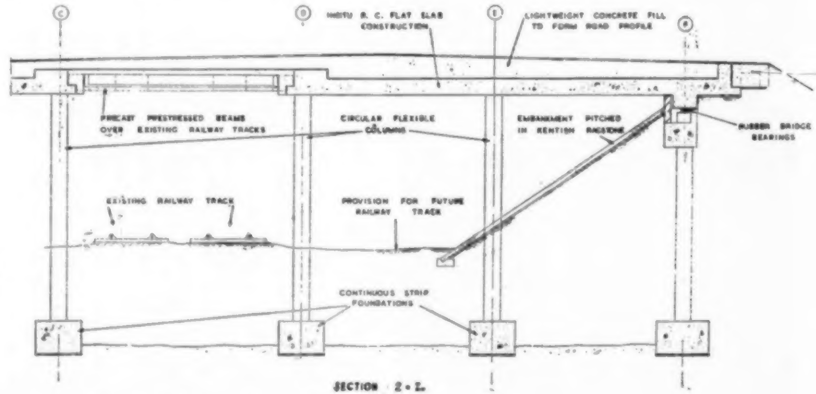


Fig. 6.—Details of Bridge shown in Fig. 7.

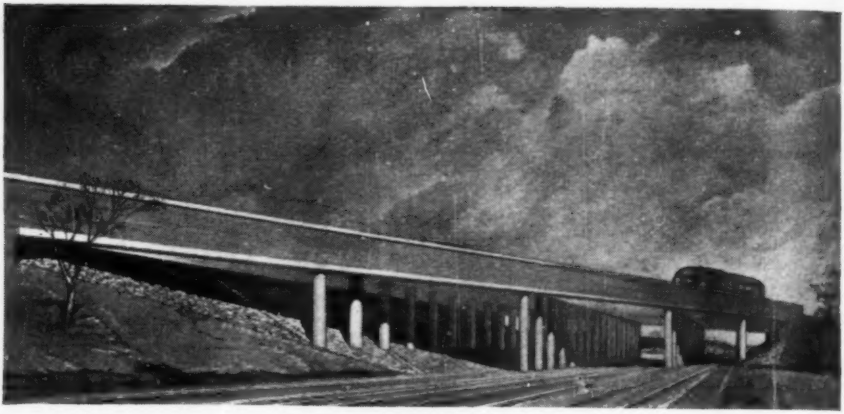


Fig. 7.—Bridge over Railway.

a reinforced concrete deck-slab cast in place on permanent shuttering supported by the beams. At the sides of the bridge the flat slabs and precast beams will be supported by reinforced concrete and precast prestressed edge-beams which will form a solid parapet, which the Railway authorities prefer to the open type.

In all the bridges a combination of flexible columns and rubber bearings will be provided to allow for movements due to changes of temperature. This type of bearing is economical, simple to install, and has the advantage that it is equally flexible in all horizontal directions; this is a major asset in bridges with considerable skews. No earth-retaining abutments or wing-walls will be required, approach spans being provided instead. It is estimated that the cost of this type of construction will be less than that of earth-retaining abutments and wing-walls; it is considered that it will also reduce the "tunnel" effect which is particularly associated with wide bridges, as well as being æsthetically desirable.

According to preliminary indications the cost of the bridges will be from a little less than £3 to a little less than £4 per square foot of overall deck area.

The roadworks have been designed by the Kent County Council (County Surveyor Mr. E. W. H. Vallis) on behalf of the Ministry of Transport and Civil Aviation. The contractors for the whole project, including the roadworks, are Messrs. Richard Costain, Ltd.

The Cement and Concrete Association.

The Hon. L. O. Russell, O.B.E., T.D., has been appointed Director-General of the Cement and Concrete Association in succession to Sir Francis Meynell, R.D.I., who has retired. Mr. Russell was Director of the British Institute of Management from 1947 to 1956. During the war he served in the Army, reaching the rank of Brigadier, and took part in the planning of the Normandy landing. On leaving the Army he became an Assistant Secretary in the Board of Trade.

Proposed Tunnel under the Tyne.

THE Ministry of Transport and Civil Aviation states that it is hoped to authorise in the year 1958–1959 the construction of a tunnel for road vehicles under the River Tyne at a cost of about £13,000,000. The consulting engineers are Messrs. Mott, Hay, and Anderson.

The Steenbras Dam.

A FILM illustrating the strengthening and raising of the Steenbras dam, near Capetown, may be obtained on loan from the Cementation Co., Ltd. (20 Albert Embankment, London, S.E.11).

Design of Eccentrically-Loaded Columns by the Load-Factor Method.

4.—Unsymmetrical Mild Steel Reinforcement.*

By J. D. BENNETT, B.Eng.

THE notation and assumptions on which the accompanying charts are based are given in the first article of this series (see this journal for November 1957). The analysis is similar to that used for the charts for unsymmetrical cold-worked bars. The charts are simple to use, contain all the information required for the design of columns, and give the total area of steel required and the proportions to be used near each face.

From the linear strain relationship at balanced failure, $\dfrac{2p_{st}}{E_s(1-n)} = \dfrac{1}{300n}$.

Assuming that $E_s = 30 \times 10^6$ lb. per square inch, $n = \dfrac{100{,}000}{100{,}000 + 2p_{st}}$.

For mild steel, $p_{st} = 20{,}000$ lb. per square inch, which gives $n = 0{\cdot}7143$ for balanced failure. For values of n greater than 0·7143 failure will be due to compression with $n = \dfrac{100{,}000}{100{,}000 + 2p_t}$; p_t is less than p_{st}, and $2p_t$ is the stress in the tensile steel at ultimate load. For values of n less than 0·7143 failure will be due to yielding of the tensile steel, and the stress in the tensile steel at ultimate load will be the yield stress $2p_{st}$.

Failure in Tension.

For any distribution of steel the equations of equilibrium will be as follows.

Resolving forces axially, $\dfrac{P}{bdp_{cc}} = 0{\cdot}85\dfrac{d_1}{d}n - 20{,}000\dfrac{r_1}{p_{cc}} + 18{,}000\dfrac{r_2}{p_{cc}}$.

Using the relationship $r_1 + r_2 = r$, this can be written in the form

$$\frac{P}{bdp_{cc}} = 0{\cdot}85\frac{d_1}{d}n - 20{,}000\frac{r}{p_{cc}} + 38{,}000\frac{r_2}{p_{cc}} \quad . \quad . \quad . \quad (1)$$

Calculating moments about the tensile steel,

$$\frac{M}{bd^2p_{cc}} = 0{\cdot}85\left(\frac{d_1}{d}\right)^2 n - \frac{0{\cdot}85^2}{2}\left(\frac{d_1}{d}\right)^2 n^2$$
$$+ 18{,}000\left(\frac{d_1 - d_2}{d}\right)\frac{r_2}{p_{cc}} - \left(\frac{d_1 - d_2}{2d}\right)\frac{P}{bdp_{cc}} \quad . \quad (2)$$

Substituting $\dfrac{r_2}{p_{cc}}$ from (1), $\dfrac{M}{bd^2p_{cc}} = 0{\cdot}85\left(\dfrac{d_1}{d}\right)^2 n - \dfrac{0{\cdot}85^2}{2}\left(\dfrac{d_1}{d}\right)^2 n^2$

$$+ \frac{18}{38}\left(\frac{d_1 - d_2}{d}\right)\left[\frac{P}{bdp_{cc}} - 0{\cdot}85\frac{d_1}{d}n + 20{,}000\frac{r}{p_{cc}}\right] - \left(\frac{d_1 - d_2}{2d}\right)\frac{P}{bdp_{cc}} \quad . \quad (3)$$

* Concluded from November and December, 1957, and March, 1958.

For a particular load P and ratio of reinforcement $\frac{r}{p_{cc}}$ there will be three variables, namely M, n, and r_2. The relation between n and r_2 is given by equation (1) and this is included in equation (3), which can be differentiated to determine the value of n which gives a maximum value of M :

$$\frac{1}{bd^2p_{cc}}\cdot\frac{\partial M}{\partial n} = 0{\cdot}85\left(\frac{d_1}{d}\right)^2 - 0{\cdot}85^2\left(\frac{d_1}{d}\right)^2 n - \frac{18}{38}\left(\frac{d_1 - d_2}{d}\right) \times 0{\cdot}85\frac{d_1}{d} = 0.$$

Hence
$$n = \frac{\frac{d_1}{d} - \frac{18}{38}\left(\frac{d_1 - d_2}{d}\right)}{0{\cdot}85\frac{d_1}{d}}.$$

Values of n corresponding to various values of $\frac{d_1}{d}$ are as follows.

$\frac{d_1}{d}$:	0·95	0·90	0·85	0·80	0·75
n:	0·6485	0·6810	0·7175	0·7585	0·8049

Thus, when $\frac{d_1}{d}$ is 0·95 or 0·90 the value of n shown in the foregoing gives the maximum value of M ; this value is used for the range of tensile failure where r_2 [obtained from equation (1)] varies between 0 and r. When r_2 is zero there is no steel in the compressive face and the equations of equilibrium become

$$\frac{P}{bdp_{cc}} = 0{\cdot}85\frac{d_1}{d}n - 20{,}000\frac{r}{p_{cc}} \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{M}{bd^2p_{cc}} = 0{\cdot}85\left(\frac{d_1}{d}\right)^2 n - \frac{0{\cdot}85^2}{2}\left(\frac{d_1}{d}\right)^2 n^2 - \left(\frac{d_1 - d_2}{2d}\right)\frac{P}{bdp_{cc}} \quad . \quad (5)$$

When r_2 is equal to r there is no steel in the tensile face and the equations of equilibrium become

$$\frac{P}{bdp_{cc}} = 0{\cdot}85\frac{d_1}{d}n + 18{,}000\frac{r}{p_{cc}} \quad . \quad . \quad . \quad . \quad (6)$$

$$\frac{M}{bd^2p_{cc}} = 0{\cdot}85\left(\frac{d_1}{d}\right)^2 n - \frac{0{\cdot}85^2}{2}\left(\frac{d_1}{d}\right)^2 n^2 + 18{,}000\left(\frac{d_1 - d_2}{d}\right)\frac{r}{p_{cc}} - \left(\frac{d_1 - d_2}{2d}\right)\frac{P}{bdp_{cc}} \quad . \quad (7)$$

Equations (4) to (7) are used for plotting $\frac{P}{bdp_{cc}}$ against $\frac{M}{bd^2p_{cc}}$, with values of n between 0 and 0·7143, for the ranges not covered by equations (1) and (2).

When $\frac{d_1}{d}$ is 0·85, 0·80, or 0·75, the values of n giving maximum M in equation (2) are greater than 0·7143 and are therefore inadmissible for failure due to tension. For these values of $\frac{d_1}{d}$ the maximum possible value of n in equation (2)

is 0·7143, and this value is used to give a range of failure for "balanced" designs where r_2 varies between 0 and r. When r_2 is zero there is no steel in the compressive face; equations (4) and (5) are then used. The upper limit of r_2 is determined by the point at which compressive failure occurs. *Figs.* (1) and (2) illustrate the various types of failure.

Failure in Compression.

When n is greater than 0·7143 failure is due to the crushing of the concrete at the compressive face and the maximum strain of the concrete is $\frac{1}{300}$. From the linear strain relationship the strain in the tensile steel at ultimate load will be $\frac{1-n}{300n}$ and, since mild steel can be assumed to have a modulus of elasticity of 30×10^6 lb. per square inch for all stresses up to 40,000 lb. per square inch, the stress in the steel is given by

$$2p_t = 30 \times 10^6 \times \frac{1-n}{300n}, \text{ or } p_t = \frac{50{,}000}{n} - 50{,}000 \quad . \quad . \quad (8)$$

The equations of equilibrium are as follows. Resolving forces axially,

$$\frac{P}{bdp_{cc}} = 0{\cdot}85\frac{d_1}{d}n + 18{,}000\frac{r_2}{p_{cc}} - \left(\frac{50{,}000}{n} - 50{,}000\right)\frac{r_1}{p_{cc}}.$$

By using the relationship $r_1 + r_2 = r$ this can be written

$$\frac{r_1}{p_{cc}} = \frac{0{\cdot}85\frac{d_1}{d}n + 18{,}000\frac{r}{p_{cc}} - \frac{P}{bdp_{cc}}}{\frac{50{,}000}{n} - 32{,}000} \quad . \quad . \quad . \quad (9)$$

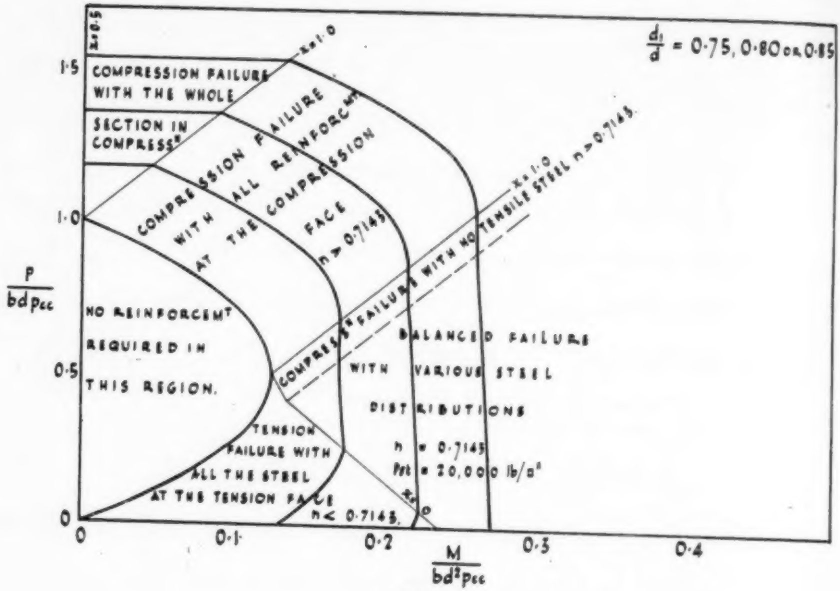
Calculating moments about the tensile steel,

$$\frac{M}{bd^2p_{cc}} = 0{\cdot}85\left(\frac{d_1}{d}\right)^2 n - \frac{0{\cdot}85^2}{2}\left(\frac{d_1}{d}\right)^2 n^2$$
$$+ 18{,}000\left(\frac{d_1 - d_2}{d}\right)\frac{r_2}{p_{cc}} - \left(\frac{d_1 - d_2}{2d}\right)\frac{P}{bdp_{cc}} \quad . \quad (10)$$

By using equation (9) this can be rewritten as

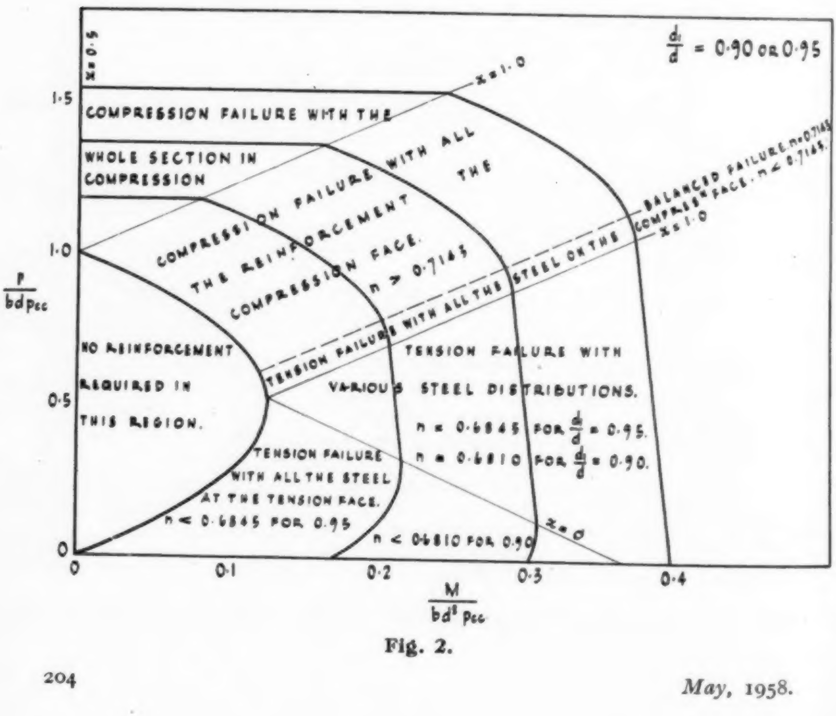
$$\frac{M}{bd^2p_{cc}} = 0{\cdot}85\left(\frac{d_1}{d}\right)^2 n - \frac{0{\cdot}85^2}{2}\left(\frac{d_1}{d}\right)^2 n^2 + 18{,}000\left(\frac{d_1 - d_2}{d}\right)\frac{r}{p_{cc}}$$
$$- 18{,}000\left(\frac{d_1 - d_2}{d}\right)\left[\frac{0{\cdot}85\frac{d_1}{d}n + 18{,}000\frac{r}{p_{cc}} - \frac{P}{bdp_{cc}}}{\frac{50{,}000}{n} - 32{,}000}\right] - \left(\frac{d_1 - d_2}{2d}\right)\frac{P}{bdp_{cc}} \quad . \quad (11)$$

For a particular load P and ratio of reinforcement $\frac{r}{p_{cc}}$ there will be three



d₁/d = 0·75, 0·80 or 0·85
x = 0·5
x = 1·0
COMPRESSION FAILURE WITH THE WHOLE SECTION IN COMPRESS^{n}
COMPRESSION FAILURE WITH ALL REINFORC^{MT} AT THE COMPRESSION FACE
n > 0·7143
COMPRESS^{N} FAILURE WITH NO TENSILE STEEL n > 0·7143
NO REINFORCEM^{T} REQUIRED IN THIS REGION.
BALANCED FAILURE WITH VARIOUS STEEL DISTRIBUTIONS
n = 0·7143
Pst = 20,000 lb/in²
TENSION FAILURE WITH ALL THE STEEL AT THE TENSION FACE
n < 0·7143.
x = 0
P / bd pcc
M / bd² pcc

Fig. 1.



d₁/d = 0·90 or 0·95
x = 0·5
x = 1·0
COMPRESSION FAILURE WITH THE WHOLE SECTION IN COMPRESSION
COMPRESSION FAILURE WITH ALL THE REINFORCEMENT THE COMPRESSION FACE.
n > 0·7143
BALANCED FAILURE n = 0·7143
COMPRESS^{N} FACE. n < 0·7143.
TENSION FAILURE WITH ALL THE STEEL ON THE
NO REINFORCEMENT REQUIRED IN THIS REGION.
TENSION FAILURE WITH VARIOUS STEEL DISTRIBUTIONS.
n = 0·6845 FOR d₁/d = 0·95.
n = 0·6810 FOR d₁/d = 0·90.
TENSION FAILURE WITH ALL THE STEEL AT THE TENSION FACE.
n < 0·6845 FOR 0·95
n < 0·6810 FOR 0·90
x = 0
P / bd pcc
M / bd² pcc

Fig. 2.

variables, namely M, n, and r_1. The relationship between n and r_1 is given by equation (9) and is included in equation (11), which can be differentiated to determine the value of n which gives a maximum value of M:

$$\frac{1}{bd^2p_{cc}}\cdot\frac{\partial M}{\partial n} = 0{\cdot}85\left(\frac{d_1}{d}\right)^2 - 0{\cdot}85^2\left(\frac{d_1}{d}\right)^2 n - 18{,}000\left(\frac{d_1-d_2}{d}\right)$$

$$\times\left[\frac{0{\cdot}85\frac{d_1}{d}\left(\frac{50{,}000}{n}-32{,}000\right)+\frac{50{,}000}{n^2}\left(0{\cdot}85\frac{d_1}{d}n+18{,}000\frac{r}{p_{cc}}-\frac{P}{bdp_{cc}}\right)}{\left(\frac{50{,}000}{n}-32{,}000\right)^2}\right]=0$$

which reduces to

$$740\left(\frac{d_1}{d}\right)^2 n^3 - \left[4161\left(\frac{d_1}{d}\right)^2 - 490\frac{d_1}{d}\right]n^2 + \left[7586\left(\frac{d_1}{d}\right)^2 - 1530\frac{d_1}{d}\right]n$$

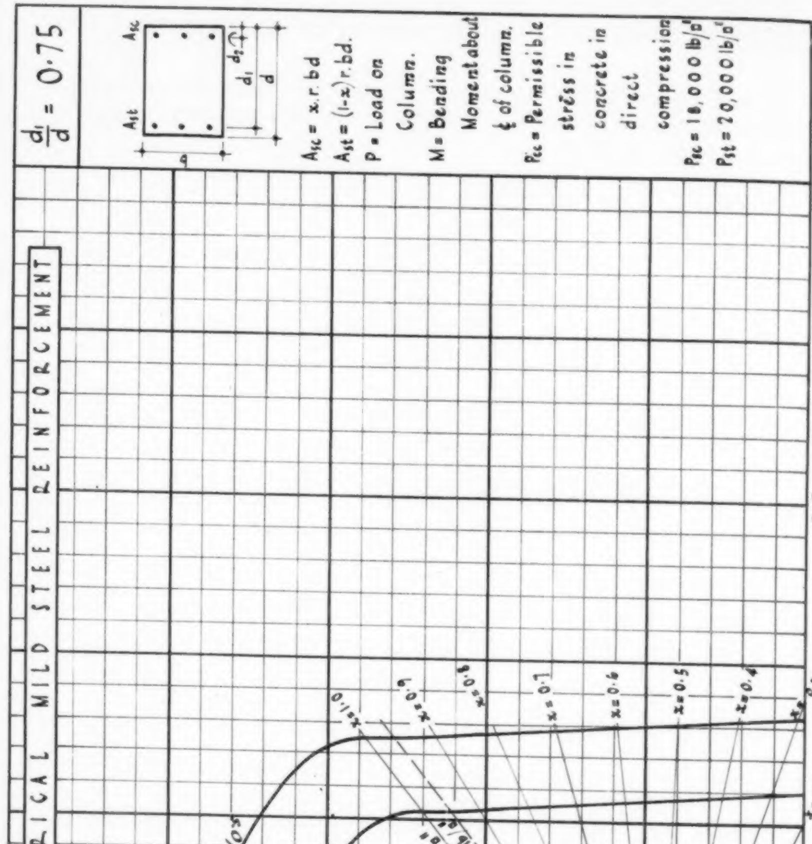
$$- 2125\left(\frac{d_1}{d}\right)^2 + 900\left(\frac{d_1-d_2}{d}\right)\left(18{,}000\frac{r}{p_{cc}} - \frac{P}{bdp_{cc}}\right) = 0 \quad . \quad (12)$$

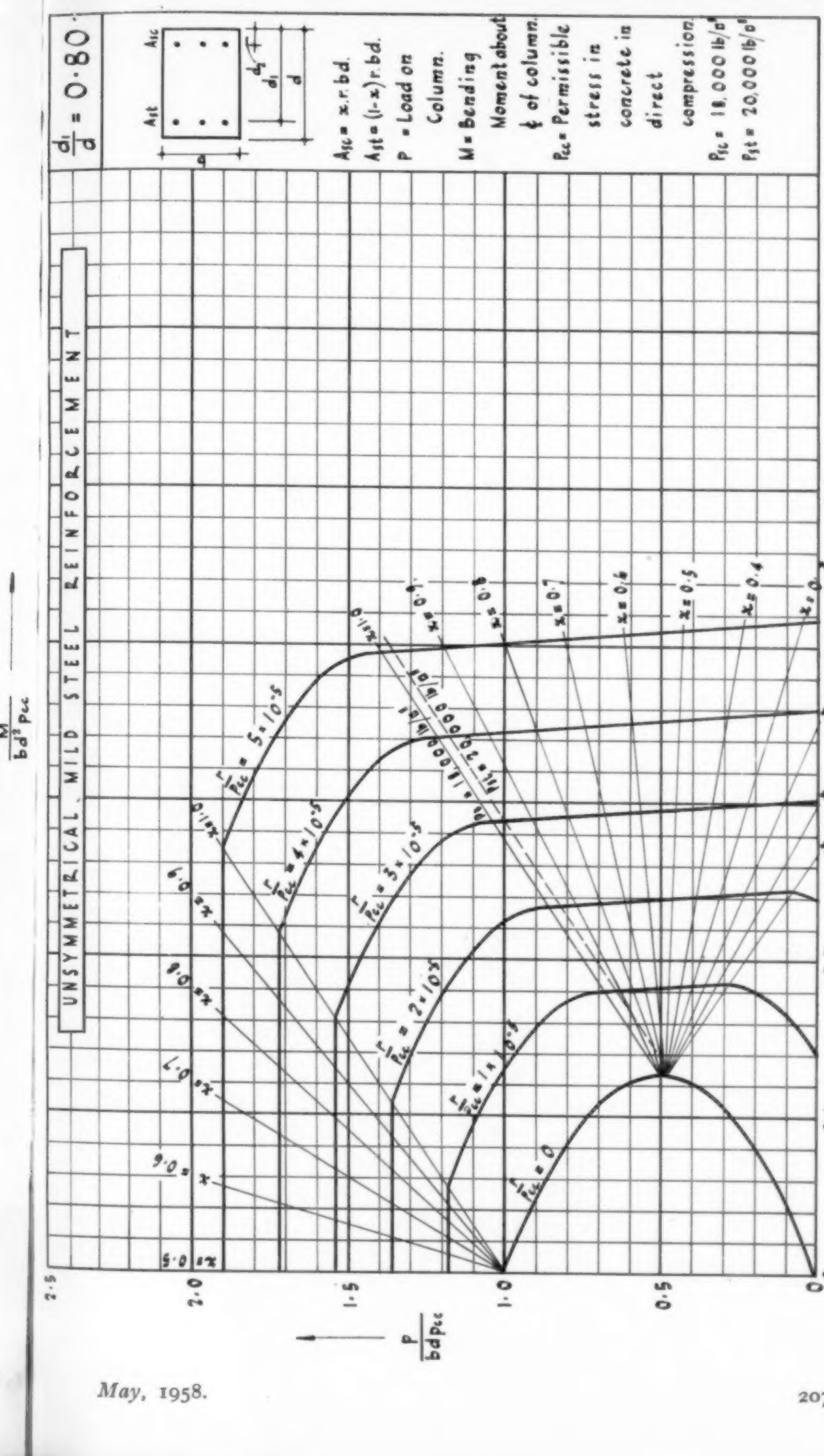
Equation (12) is valid for values for n greater than 0·7143 which, when substituted in (9), give values of r_1 between 0 and r. When $\frac{d_1}{d}$ is 0·95, 0·90, or 0·85, these two conditions cannot both be satisfied, since, when $n = 0{\cdot}7143$, equation (9) gives a negative value for r_1, which is obviously inadmissible. When $\frac{d_1}{d}$ is equal to 0·80 or 0·75 there is a small region in which compressive failure occurs with tensile reinforcement in the section, and equation (12) applies. For particular values of $\frac{P}{bdp_{cc}}$ and $\frac{r}{p_{cc}}$ equation (12) is solved for n, and the value obtained is used in (9) and (10) to determine $\frac{r_1}{p_{cc}}$ and $\frac{M}{bd^2p_{cc}}$. Elsewhere compressive failure occurs with all the steel in compression, and equations (6) and (7) apply when there is no steel in the tensile face. These equations are valid until $n = \frac{1}{0{\cdot}85\frac{d_1}{d}}$, when all the concrete is in compression and the bending moment is resisted by the total area of the steel placed at the compressive face. This is the greatest value of $\frac{P}{bdp_{cc}}$ which can be obtained, and for smaller bending moments the distribution of the compressive steel must be more uniform, as given by (13) and (14).

Resolving forces axially,
$$\frac{P}{bdp_{cc}} = 1 + 18{,}000\frac{r}{p_{cc}} \quad . \quad . \quad . \quad . \quad (13)$$

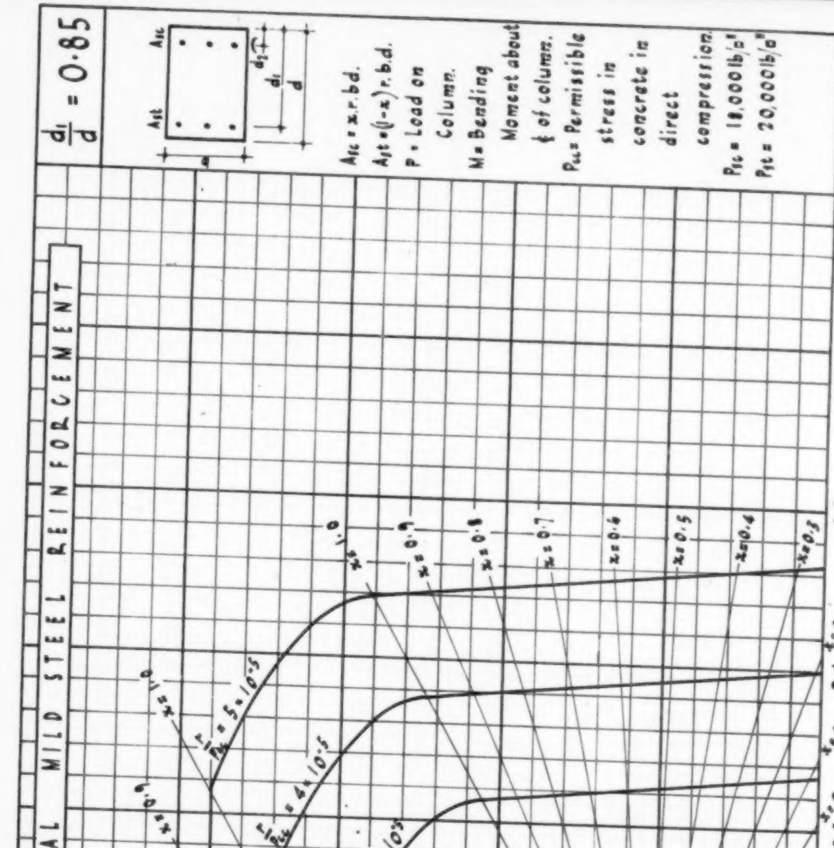
Calculating moments about the centre-line of the column,
$$\frac{M}{bd^2p_{cc}} = 18{,}000\left(\frac{r_2}{p_{cc}} - \frac{r_1}{p_{cc}}\right)\left(\frac{d_1-d_2}{2d}\right) \quad . \quad . \quad . \quad (14)$$

From (13) it can be seen that $\frac{P}{bdp_{cc}}$ is constant for a given value of $\frac{r}{p_{cc}}$, and

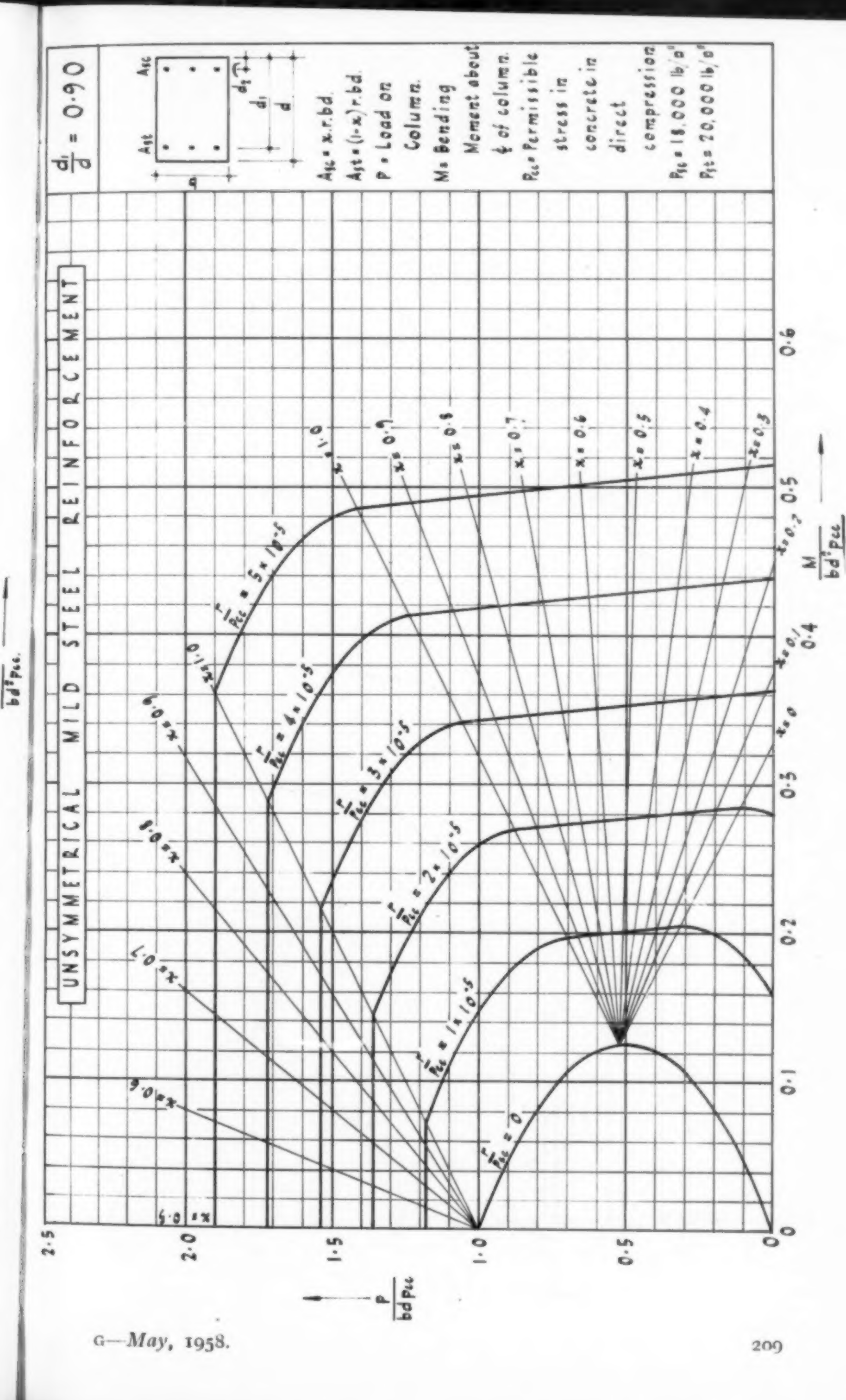

UNSYMMETRICAL MILD STEEL REINFORCEMENT
d1/d = 0·75
Asc = x.r.bd
Ast = (1-x)r.bd.
P = Load on Column.
M = Bending Moment about ℄ of column.
Pcc = Permissible stress in concrete in direct compression
Pcc = 18,000 lb/□"
Pst = 20,000 lb/□"
r/Pcc = 5×10⁻⁵
r/Pcc = 4×10⁻⁵
r/Pcc = 3×10⁻⁵
r/Pcc = 2×10⁻⁵
r/Pcc = 1×10⁻⁵
r/Pcc = 0
P/bdPcc
M/bd²Pcc
0
0·1
0·2
0·3
0·4
0·5
0·5
1·0
1·5
2·0
2·5
x=0
x=0·1
x=0·2
x=0·3
x=0·4
x=0·5
x=0·6
x=0·7
x=0·8
x=0·9
x=1·0

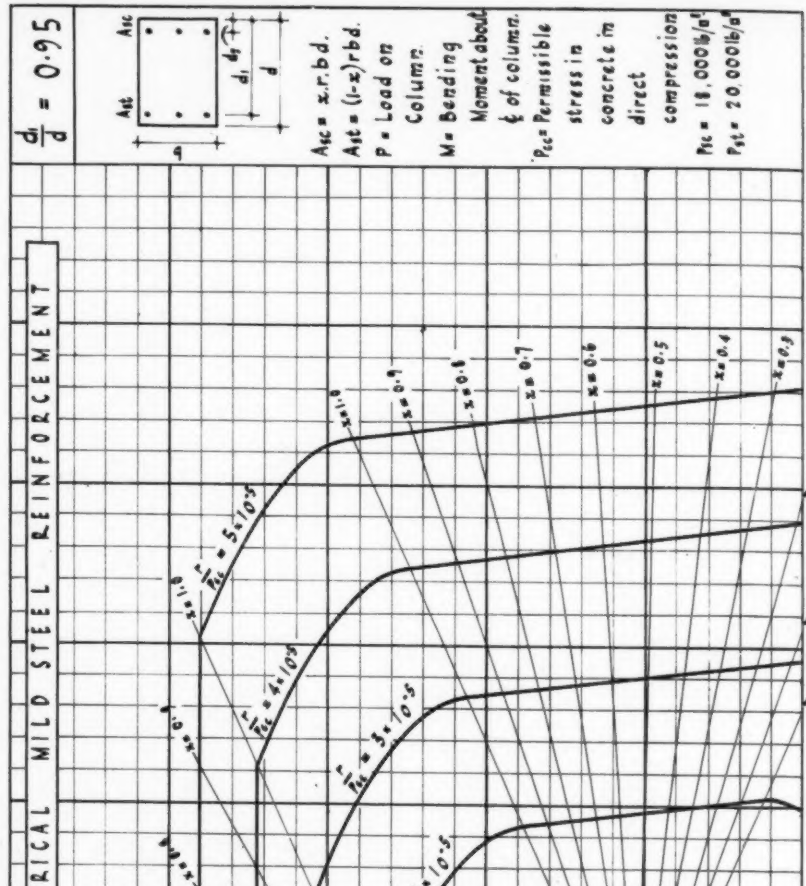


UNSYMMETRICAL MILD STEEL REINFORCEMENT
$\frac{d_1}{d} = 0\cdot80$
Ast Asc
b
d_2 d_1 d
A_{sc} = x.r.bd.
A_{st} = (1-x) r.bd.
P = Load on Column.
M = Bending Moment about ℄ of column.
p_{cc} = Permissible stress in concrete in direct compression.
p_{sc} = 18,000 lb/□"
p_{st} = 20,000 lb/□"
$\frac{r}{p_{cc}} = 0$
$\frac{r}{p_{cc}} = 1 \times 10^{-5}$
$\frac{r}{p_{cc}} = 2 \times 10^{-5}$
$\frac{r}{p_{cc}} = 3 \times 10^{-5}$
$\frac{r}{p_{cc}} = 4 \times 10^{-5}$
$\frac{r}{p_{cc}} = 5 \times 10^{-5}$
p_{sc} = 18,000 lb/□"
p_{st} = 20,000 lb/□"
x = 0.5
x = 0.6
x = 0.7
x = 0.8
x = 0.9
x = 1.0
x = 1.0
x = 0.9
x = 0.8
x = 0.7
x = 0.6
x = 0.5
x = 0.4
x = 0.3
x = 0.2
x = 0.1
x = 0
$\frac{P}{bd\,p_{cc}}$
$\frac{M}{bd^2 p_{cc}}$
0
0.1
0.2
0.3
0.4
0.5
0.5
1.0
1.5
2.0
2.5



UNSYMMETRICAL MILD STEEL REINFORCEMENT
$\frac{d_1}{d} = 0.85$
A_{sc}
A_{st}
d_1
d_2
d
$A_{sc} = x.r.bd.$
$A_{st} = (1-x).r.b.d.$
P = Load on Column.
M = Bending Moment about ℄ of column.
p_{cc} = Permissible stress in concrete in direct compression.
p_{cc} = 18,000 lb/□"
p_{st} = 20,000 lb/□"
$\frac{P}{bdp_{cc}}$
$\frac{M}{bd^2p_{cc}}$
$\frac{r}{p_{cc}} = 0.$
$\frac{r}{p_{cc}} = 1 \times 10^{-5}$
$\frac{r}{p_{cc}} = 2 \times 10^{-5}$
$\frac{r}{p_{cc}} = 3 \times 10^{-5}$
$\frac{r}{p_{cc}} = 4 \times 10^{-5}$
$\frac{r}{p_{cc}} = 5 \times 10^{-5}$
x = 0
x = 0.1
x = 0.2
x = 0.3
x = 0.4
x = 0.5
x = 0.6
x = 0.7
x = 0.8
x = 0.9
x = 1.0


UNSYMMETRICAL MILD STEEL REINFORCEMENT
$\frac{d_1}{d} = 0{\cdot}90$
Asc
Ast
b
d_2
d_1
d
Asc = x.r.bd.
Ast = (1-x).r.bd.
P = Load on Column.
M = Bending Moment about ℄ of column.
Pcc = Permissible stress in concrete in direct compression
Psc = 18,000 lb/□"
Pst = 20,000 lb/□"
$\frac{r}{p_{cc}} = 0$
$\frac{r}{p_{cc}} = 1 \times 10^{-5}$
$\frac{r}{p_{cc}} = 2 \times 10^{-5}$
$\frac{r}{p_{cc}} = 3 \times 10^{-5}$
$\frac{r}{p_{cc}} = 4 \times 10^{-5}$
$\frac{r}{p_{cc}} = 5 \times 10^{-5}$
x = 0
x = 0.1
x = 0.2
x = 0.3
x = 0.4
x = 0.5
x = 0.6
x = 0.7
x = 0.8
x = 0.9
x = 1.0
$\frac{M}{bd^2 p_{cc}}$
$\frac{P}{bd\,p_{cc}}$
0
0·1
0·2
0·3
0·4
0·5
0·6
0·5
1·0
1·5
2·0
2·5



UNSYMMETRICAL MILD STEEL REINFORCEMENT
d1/d = 0·95
Asc
Ast
b
d1
d
Asc = x.r.b.d.
Ast = (1-x)r.b.d.
P = Load on Column.
M = Bending Moment about ℄ of column.
Pcc = Permissible stress in concrete in direct compression
Psc = 18,000 lb/□"
Pst = 20,000 lb/□"
r/Pcc = 0
r/Pcc = 1×10⁻⁵
r/Pcc = 2×10⁻⁵
r/Pcc = 3×10⁻⁵
r/Pcc = 4×10⁻⁵
r/Pcc = 5×10⁻⁵
x = 1.0
x = 0.9
x = 0.8
x = 0.7
x = 0.6
x = 0.5
x = 0.4
x = 0.3
x = 0.2
x = 0.1
x = 0
M/bd²Pcc
P/bd Pcc
0
0·1
0·2
0·3
0·4
0·5
0·6
0·5
1·0
1·5
2·0
2·5

(14) shows how the bending moment is resisted by the eccentricity of the compressive steel.

Examples.

The example used in the first article is now considered again. A column 24 in. by 12 in. in cross section, with 1 : 2 : 4 concrete, and $p_{sc} = 760$ lb. per square inch, supports a direct load P of 130,000 lb. and a bending moment M of 1,300,000 in.-lb. Then

$$\frac{P}{bdp_{cc}} = \frac{130{,}000}{12 \times 24 \times 760} = 0{\cdot}595, \text{ and } \frac{M}{bd^2p_{cc}} = \frac{1{,}300{,}000}{12 \times 24^2 \times 760} = 0{\cdot}247.$$

Assuming that $\frac{d_1}{d} = 0{\cdot}90$, the graph indicates that $\frac{r}{p_{cc}} = 1{\cdot}63 \times 10^{-5}$ and $x = 0{\cdot}64$.

Therefore $A_{sc} = 0{\cdot}64 \times 1{\cdot}63 \times 10^{-5} \times 760 \times 12 \times 24 = 2{\cdot}28$ sq. in.
and $A_{st} = 0{\cdot}36 \times 1{\cdot}63 \times 10^{-5} \times 760 \times 12 \times 24 = 1{\cdot}28$ sq. in.

This gives a total area of steel of 3·56 sq. in., and the area required for a symmetrically-reinforced column would be 3·98 sq. in.

In *Table* I the different methods of reinforcing this column for the loads given in the example and also for a more extreme loading case are compared.

TABLE I.

Loading	Symmetrical cold-worked bars. Total area	Symmetrical mild steel. Total area	Unsymmetrical cold-worked bars			Unsymmetrical mild steel		
			A_{st}	A_{sc}	Total area	A_{st}	A_{sc}	Total area
$M = 1{,}300{,}000$ in.-lb. $P = 130{,}000$ lb.	3·28	3·98	0·84	2·04	2·88	1·28	2·28	3·56
$M = 1{,}400{,}000$ in.-lb. $P = 40{,}000$ lb.	3·61	5·32	2·27	0·82	3·09	3·82	0·10	3·92

[In the charts given in these articles tensile failure occurs in the regions below the lines of permissible stress in the steel for 30,000 and 20,000 lb. per square inch, and the stress in the steel is constant at 30,000 lb. per square inch for cold-worked bars or 20,000 lb. per square inch for mild steel bars. In all the charts the whole area can be used, and the stresses in the steel are always within the permissible values if linear interpolation is used between the lines for $\frac{r}{p_{cc}}$ to determine the percentage of reinforcement required.]

(Concluded.)

Rapid Construction of a Cooling Tower.

By the use of an unusual form of access scaffold a hyperbolic cooling tower at Castle Donnington power station, near Derby, has been built in twenty weeks. The tower has a height of 310 ft., a diameter at the base of 212 ft., a least diameter of 117 ft. 6 in., and a diameter at the top of 120 ft.

Tubular steel scaffolding (*Fig.* 1) of 2 in. diameter was fixed to the inner face of the reinforced concrete wall by means of special fittings as shown in *Fig.* 2. The scaffold was constantly maintained at up to three lifts ahead of the concreting. The scaffold was erected by ten men, and is estimated to have required little more than half the amount of tubing that would have been used for an ordinary "birdcage" scaffold. The cost was between 50 and 60 per cent. of that for previous towers.

The scaffold was tied to the wall by 6-in. by 2-in. steel plates, to each of which was tack-welded a $\frac{5}{8}$-in. Whitworth nut. Into each nut was screwed a 6-in. bolt the shank of which was protected by a cardboard sleeve $1\frac{7}{8}$ in. long and $\frac{5}{8}$ in. diameter. The completed assemblies were placed equidistantly around the steel shuttering prior to casting a section of concrete, the bolt-heads protruding through holes in the shutter plates. As the shutters were 3 ft. 6 in. deep it was necessary to insert ties in alternate courses only, thus providing working lifts of 7 ft.

After the concrete had been placed and the shuttering removed, the steel fixing plates were firmly embedded in the concrete and the bolts were removed or replaced in the cardboard tubes as required. The scaffolding was then anchored at these points by half a standard swivel-clip to which was riveted a mild steel right-angle bracket. One end of the clip was clamped over the scaffold tube, and the bracket tightened against the face of the concrete by the bolt. As the plates were inserted behind the reinforcement the load could be applied immediately on removal of the shutters, even if the concrete were still uncured and green.

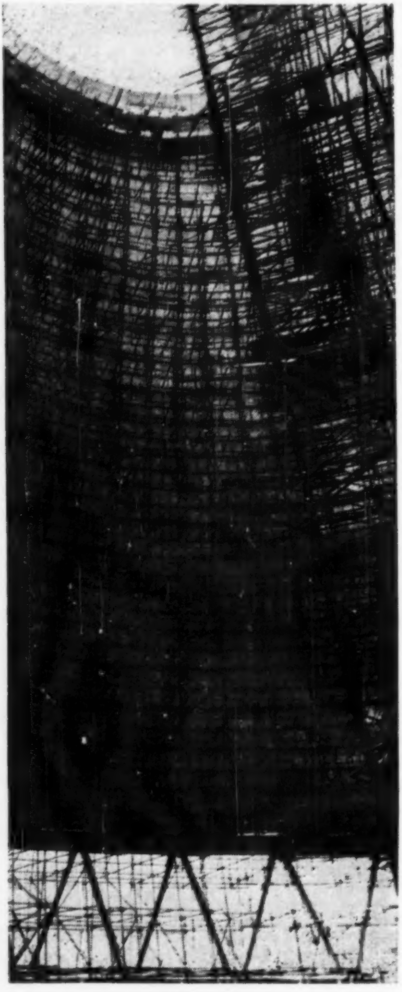


Fig. 1.

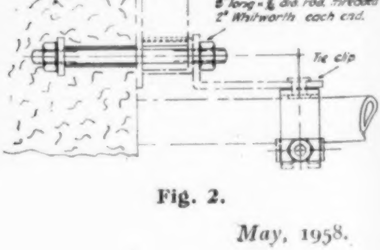


Fig. 2.

When the scaffold was stripped the holes left in the concrete were plugged with cement. The contractors were the Mitchell Construction Co., Ltd., and the scaffold was designed and erected by Mills Scaffold Co., Ltd.

Book Reviews.

"**Distribution of Deformation.**" By C. V. Kloucek. (London: Constable & Co., Ltd. 1955. Price 50*s*.)

PREVIOUSLY available in a limited English edition only, and consequently relatively unknown, the present edition of this book (printed in the English language) describing a method of structural analysis akin to slope-deflection but not requiring the solution of simultaneous equations may be welcomed by those concerned with multiple-bay multiple-story structures. A description of the method was published in this journal for April, 1949. Briefly it consists in determining the rotation of a joint due to loads on adjacent members and adding to this the secondary rotations carried over from rotations of adjacent joints. For continuous beams the method appears to be neither better nor worse than most others. However, the value of the book lies in the second part where for multiple-story structures subject to sway the concept of the vertical cantilever of stiffness equivalent to that of the entire frame is introduced; the angular deformation of this cantilever at the level of each story is roughly equal to the rotation of the joints of the frame at the same level, and this value is used as the first step in an iterative solution of the slope-deflection equations for the frame. In general sufficient accuracy is given by the second iteration. The method is rapid even for complicated frames.

—J. E. G.

"**The Chapel at Ronchamp.**" By Le Corbusier. (London: Architectural Press, Ltd. Price 25*s*.)

IT is doubtful if so many magnificent photographs have ever before been published of one small building as are given here of a chapel designed by Le Corbusier and built on a hill-top in France. The book will delight those who like architecture that used to be called modernistic or futurist and is now contemporary, and who will enthuse about the beauty of what may appear to others to be meaningless lumps of concrete scattered about the building, and the loopholes in walls made immensely thick so that the holes will admit beams of sunlight in particular directions at particular times on particular days of the year. To some it may seem unusual for an architect to proclaim his own work as "great music", and ingenuous to tell his readers that they will "discover the game" if they look at the photographs upside down or sideways. Others may wonder whether they are reading aright a statement that the smooth saucer-shaped roof was inspired by a knobby crab-shell picked up on Long Island near New York and which is honoured with a photograph in the book. The publishers describe the author as a genius—and that must be sufficient excuse for those who find difficulty in understanding much of his writing—the first sentence, for example, reads: "This is what slipshod, complacent language, and what the superficial mind (the shrug of the shoulders in club and salon with its 'Well, why not?') types as, 'Baroque' or, should you prefer it, 'Baroquism'."

"**Curtain Walling.**" (London: Iliffe & Sons, Ltd. 34 pages. Price 2*s*. 6*d*.)

THIS reprint of an article from an architectural journal gives examples of proprietary curtain wall systems, details of infilling panels, and names and addresses of manufacturers and suppliers. There are also examples of designs and details from English, American, and German sources.

"**Il Costo delle Grandi Opere d'Ingegneria.**" By Eugenio Campini. (Milan: Editore Ulrico Hoepli. 1956. Price 5000 lire.)

WRITTEN in the Italian language, the main purpose of this book is to present some novel methods derived by Dr. Campini to enable comparisons to be made of the cost of civil engineering works of widely differing types. The proposals are based on the following method. First the quantities of all materials in the work are

reduced to cubic metres of "virtual concrete" by the multiplication of the actual quantities by coefficients; secondly the labour required to produce a cubic metre of "virtual concrete" is calculated; and thirdly the ratio of the cost of the work to the number of cubic metres of "virtual concrete" is determined. It is stated that the cost per cubic metre is nearly constant for each country, and in support of this figures are quoted for more than a hundred civil engineering works throughout the world, ranging in magnitude from the Panama Canal to a three-story prestressed precast building in Italy. The value of the method is stated to be that it enables these comparisons to be made between works in different countries and at different times. The numerical investigations are summarised in 60 graphs.

"The Design of Structural Members." By H. T. Jackson. (London: Architectural Press, Ltd. Price 25*s*.)

STUDENTS of architecture, for whom it is primarily intended, should find this book useful. It deals with simple structures and aims at giving all the information needed to enable a student to pass the intermediate examination of the Royal Institute of British Architects.

"Handbook of Rigging." By W. E. Rossnagel. (London: McGraw-Hill Book Co. 342 pages. Price 49*s*.)

THIS is a useful work for those concerned with the erection of scaffolding and towers for hoists and cranes. Information is given on methods of finding the centre of gravity of objects to be hoisted, on leverage and the gearing of hoists and winches, the properties of timber, types of cranes, hoists and slings, and the prevention of accidents.

"Proceedings of the International Association for Bridge and Structural Engineering." Vol. 17. 1957. (Zürich: Verlag Leeman. Price 38 D.M. or 39.35 Swiss francs.)

THIS volume contains fifteen papers, of which eight are in English and the remainder in French. The papers relating to concrete are "Impact Resistance of Prestressed Concrete Masts", by P. W. Abeles, Great Britain; "Calculation of Bridges with Overhung Girders Connected by Joints", by J. Courbon, France; "A General Theory of Deformations of Membrane Shells", by W. Flügge and F. T. Geyling, U.S.A.; "Theory of Prismatic Folded-plate Structures", by J. E. Goldberg and H. L. Leve, U.S.A.; "Aerodynamic Study of a Cooling Tower in the Form of a Hyperboloid of Revolution", by G. Golubović, France; "Influence Surfaces for Moments in Slabs Continuous over Flexible Cross Beams", by T. Kawai and B. Thürlimann, U.S.A.; "Stress and Strain in Thin Shallow Spherical Calotte Shells", by G.-A. Oravas, Venezuela; "'Tor 60' Twisted Steel Bars. Experimental Investigation", by Y. Saillard, France.

"The Gifford-Udall-C.C.L. Handbook." (London: Cable Covers, Ltd. Price 25*s*).

IN addition to a description of the Gifford-Udall system of prestressing and its applications, this volume includes chapters on the principles of prestressing, the production of high-quality concrete, and some useful tables which apply to any system.

"Sell's Building Trades List." (London: Business Dictionaries, Ltd. Price 30*s*.)

IN this classified directory of the building and allied trades are given some 40,000 names and addresses of firms engaged in or catering for the industry and a very full list of trade names.

Competition for Design of a Factory.

THE result is announced of the competition sponsored by the Cement Marketing Co., Ltd., for the design of a factory building in reinforced concrete. The conditions were given in this journal for December last.

First prize (£1000)—Architectural design: Mr. Joseph Mendleson and Mr. J. H. C. Lamb (Messrs. Joseph Mendleson & Partners); Messrs. Chamberlain & Partners (consulting engineers), Mr. Z. L. J. Woloszczuk, and Mr. R. Wilcox.

Second prize (£500)—Architectural design: Messrs. E. H. Eames, P. G. Frome, P. Drew, P. H. Saunders, and J. D. Morris; consulting engineers: Mr. Z. L. J. Woloszczuk and Mr. A. B. Szulc.

Third prize (£250)—Architectural design: Messrs. C. J. Bromley, P. S. Buckhurst, G. V. Hansen, and J. R. Peverley; consulting engineer: Mr. K. Szmidt.

A School in Lancashire.

ALLOWANCE FOR SUBSIDENCE.

A NEW Roman Catholic secondary modern school for boys at Leigh (Lancs) consists of three buildings. The main building, comprising the assembly hall, staff rooms, and classrooms, is 180 ft. long, 70 ft. wide, and 21 ft. high; the classroom building is 132 ft. long, 50 ft. wide, and 21 ft. high, and the workshop building is 92 ft. long, 75 ft. wide, and 10 ft. high.

There are twenty-three seams of coal under the district at depths which vary from 300 ft. to 6000 ft. Some have been worked out, and those accessible are being worked continuously; it is expected that mining will continue for many years. In the past sixty years the subsidence of the surface has been about 25 ft., and further subsidence of about 10 ft. is expected to occur during the next fifteen years. The programme of extraction of coal is such that no definite period of rest between the extraction of two seams can be relied upon and, as it is the policy to build only during periods of rest, special authority was granted for the building of the school whilst subsidence was occurring.

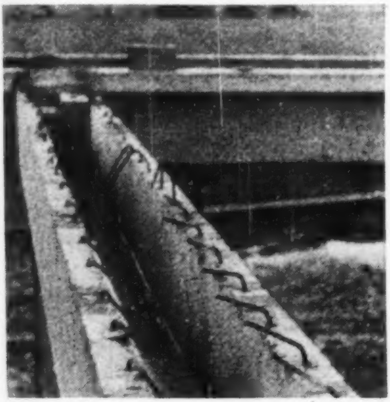


Fig. 1.—Prestressed Channel Beams.

The horizontal movement (or "draw") of the ground due to mining could not be relied upon to be always in the same direction, as the seams are being extracted in accordance with a plan which is intended to minimise disturbance of the surface drainage. The design was also affected by the extension of the coal-pillar under the nearby Bridgewater canal to include a road bridge. The site is about 450 ft. north of the canal at the nearest point, and the coal-pillar originally terminated about 1500 ft. to the west of the site. The road bridge was therefore in the area of subsidence and was rebuilt about two years ago. At the same time the pillar was extended, thereby creating a fault along the north side of the canal and a differential subsidence of about 12 in. across the site from south to north.

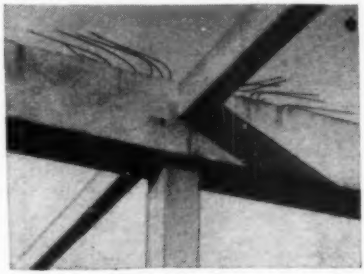


Fig. 2.—Junction of Internal Column and Beams.

Borings in the vicinity of the buildings passed through a uniform layer of clayey sand about 15 ft. thick and penetrated to a depth of 43 ft. into a layer of dense red

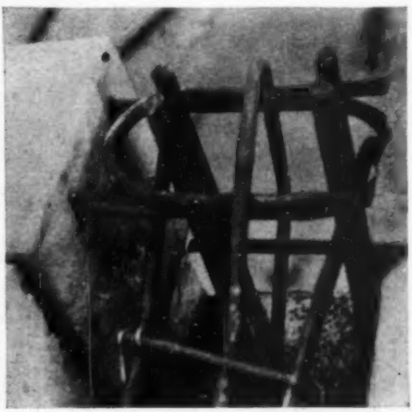


Fig. 3.—Top of External Column.

sand, the total thickness of which was not determined. Levelling was also done from a bench-mark over the coal-pillar which underlies the canal at a bridge about 900 ft. north-west of the site. The survey showed that the site had subsided by about 10 ft. since the Ordnance Survey levelling of 1926–1927; it was also discovered that the bench-mark had subsided by 0·19 ft. during this period. The extraction of coal from two seams, one 4 ft. thick and one 7 ft. thick, continued during the building of the school, and it was expected that the subsidence at the surface would be about 60 per cent. of these thicknesses.

As the uppermost layer of the ground is cohesive, it was assumed that this would balance the effect of the "draw", and would assist the structure to adapt itself to the movement. Short bored piles were used in order to reduce the resistance to the horizontal movement of the ground during the period of the "draw", and so that the friction between the piles and the clayey sand would retard and reduce the vertical settlement. The piles are reinforced to resist bending moments which may be produced by horizontal pressure of the ground.

During the period of construction, levels were taken every two months; the total average settlement was 1 ft. 6 in., the differential settlement being $7\frac{1}{2}$ in. under the workshop building and 4 in. under the main building. Subsequent measurements have shown that the differential movements have been reduced to $\frac{3}{4}$ in. under the workshop building and $\frac{5}{8}$ in. under the main building.

Factory-made precast columns and precast prestressed beams were used throughout, the slabs and stairs being cast in place. The beams are connected to the columns by pin-joints. A module of 40 in. was used in planning the buildings.

In the two-story buildings, the roofs and floors are supported by beams which consist of precast prestressed channels spanning 24 ft. filled with concrete cast in place. The channels are designed to resist loads due to construction and dead weight only; they are $2\frac{1}{2}$ in. thick, 13 in. wide, and 12 in. deep. The concrete was placed in the channels at the same time as the roof and floor slabs, and bars protruding from the precast concrete ensured a good connection (*Fig.* 1).

The construction at the top of an internal column is shown in *Fig.* 2. At the tops of the external columns the junction was made by placing stirrups in the troughs of the prestressed beams and interlocking them with bars projecting from the columns (*Fig.* 3).

The precast external roof beams are channels 12 in. deep which act as gutters

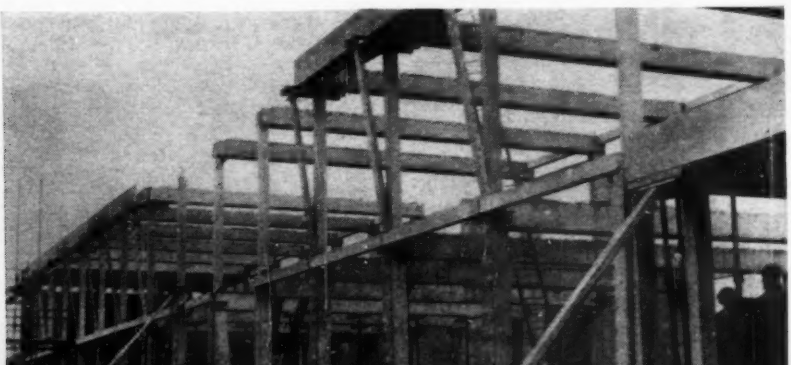


Fig. 4.—Framework of Main Building.

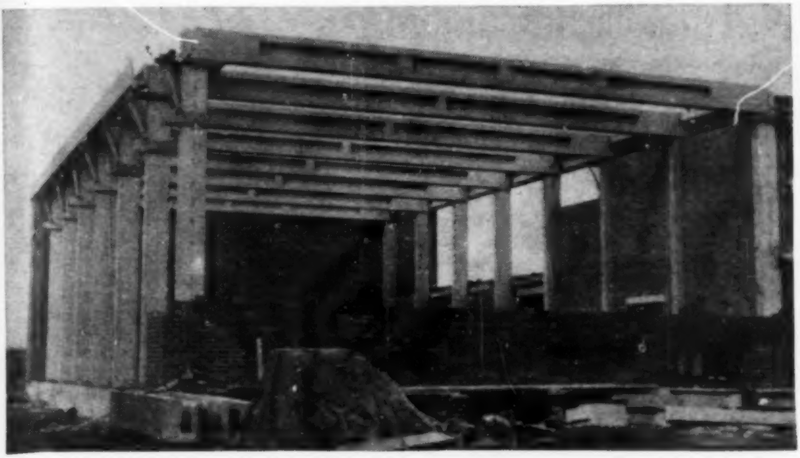


Fig. 5.—The Gymnasium during Erection.

(*Fig.* 4). As mining operations proceed the roofs are expected to drain in different directions, depending on the " draw "; the gutter units are therefore sufficiently deep to obviate the risk of overflowing.

The roofs of the single-story buildings comprise I-section prestressed precast beams, 22 in. deep and 9 in. wide, and prestressed planks 2 in. thick to which a covering was applied. The framework of the gymnasium is shown in *Fig.* 5.

The floor area of the main building is 18,535 sq. ft. and the cost of the piling, foundations, precast frames, and all concrete cast in place was £18,650. The floor area of the classroom building is 13,480 sq. ft. and the cost of similar work was £10,262. The workshop building has a floor area of 6215 sq. ft. and the cost of the roof and precast frame was £4720. As this is a single-story building, no piles were used. The total cost of the foundations and structural work was £33,632, and the total estimated cost of the school is £162,000. The structural work was completed in May, 1957.

The architects are Messrs. Weightman & Bullen, and the consulting engineers are Messrs. Taylor, Whalley & Spyra. The piling was carried out by the Cementation Co., Ltd. The general contractors were Messrs. Thomas Collier & Son, and the precast frames were supplied by the Liverpool Artificial Stone Co., Ltd.

The Volume of Construction.

THE figures issued by the Ministry of Works relating to the volume of construction carried out in each three-monthly period show that the value of such work (£537,000,000) was the same in the last quarter of the year 1957 as in the third quarter.

During the year 1957 the value of constructional work done was £2,141,000,000, compared with £2,077,000,000, in 1956. As the cost of building had increased by about 3 per cent. the volume of construction was about the same in each of these two years. The total construction in 1957 is detailed as follows (the increase or decrease compared with 1956 is given in brackets): Dwellings, £551,000,000 (– £17,000,000); new work by public authorities, £373,000,000 (+£26,000,000); new work by private developers, £457,000,000 (– £27,000,000); other new work, £830,000,000 (+ £63,000,000); repair and maintenance, £405,000,000 (+ £6,000,000); output of labour employed by public authorities, £355,000,000 (+ £12,000,000).

Tunnels Lined with Precast Concrete.

IMPROVEMENTS to the railway between London and the East Coast include three two-track tunnels, 384 yd., 232 yd., and 1214 yd. long, between New Barnet and Potters Bar. The method adopted is believed to be new.

The lining (*Fig.* 1) consists of precast concrete units with dry joints; as steam locomotives will be used for some years, the cement used for all the units (except the invert segments) and in the shafts is a Belgian metallurgical supersulphated cement which experience has shown to be able to withstand the attack of dilute sulphuric acid. The concrete has a compressive strength of about 7000 lb. per square inch at 28 days. This type of lining is particularly suited to the stiff London clay in which the tunnels are situated. Each ring is 18 in. wide and 27 in. thick, and has an internal diameter of 26 ft. 6 in.; it comprises a reinforced invert and nineteen plain precast voussoirs.

The tunnels are driven in shields of 31 ft. diameter fitted with hydraulically-operated erection arms (*Figs.* 2 and 4) which set the voussoirs into position. When the face of the excavation is sufficiently advanced, the shield is pushed forward by rams bearing against the last completed ring to allow the next ring to

Fig. 1.—Completed Tunnel.

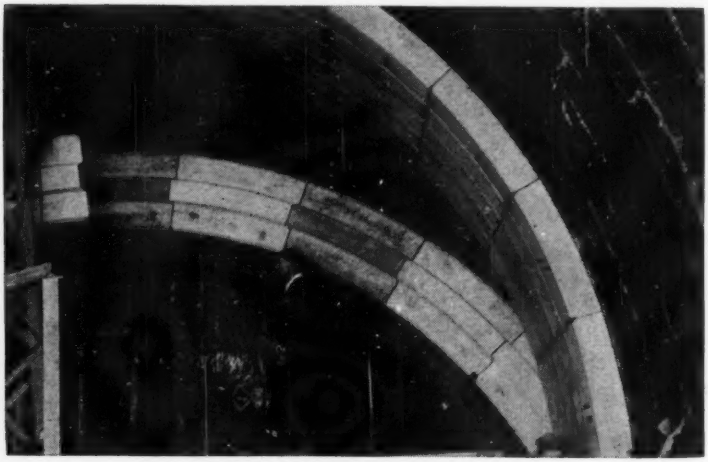


Fig. 2.—Erecting the Segments.

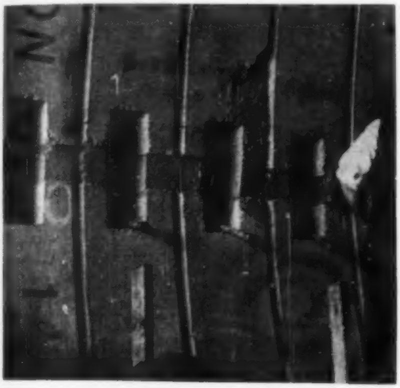


Fig. 3.—Slots for Jacks.

be assembled immediately behind the shield. The units of the ring are then forced tightly against the ground by means of jacks set in slots (*Fig.* 3) between pairs of special voussoirs at each side of the tunnel. The slots are subsequently filled with very stiff concrete, which is placed in two stages so as to avoid any release of the ground load on the ring.

Where openings occur for refuges and air-shafts special methods are adopted, including the use of concrete shear-keys between adjacent rings to support the loads on an incomplete ring. High-tensile steel rods are also incorporated.

The design of the lining is such that no fastenings are required between the rings and no grouting outside the lining. The cost of the tunnels will be less than 40 per cent. of that for cast-iron linings.

Mr. A. K. Terris is the Chief Civil Engineer of British Railways (Eastern Region). The consulting engineers for the tunnels are Sir William Halcrow & Partners, and the contractors are Messrs. Charles Brand & Son, Ltd.

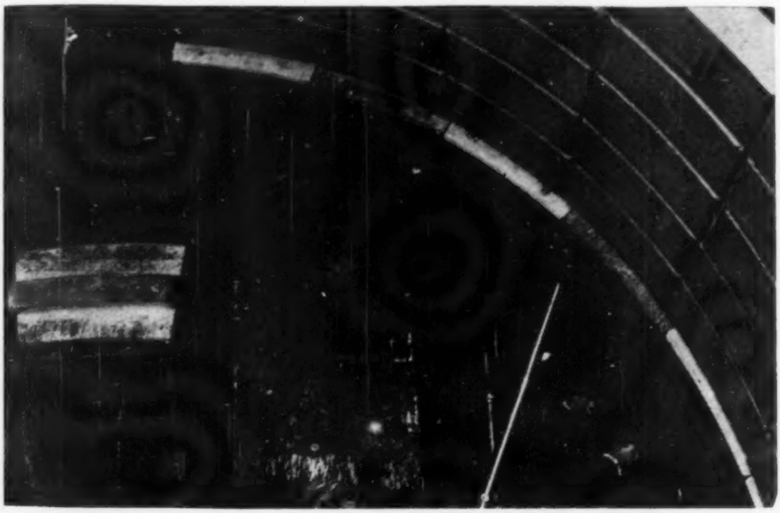


Fig. 4.—Erecting the Segments.

World Conference on Prestressed Concrete.

THE papers read at the World Conference on Prestressed Concrete, organised by the University of California and held in San Francisco in July 1957, are now available in book form. The volume is obtainable from World Conference on Prestressed Concrete, Inc., 417 Market Street, San Francisco, Calif., U.S.A., at a price of 10 dollars. The papers are as follows.

Materials and Methods.—Post-tensioning Systems and New Grouting Methods in Germany, by W. Zerna. Prestressing Steel Under High Stress, by W. O. Everling. Problems and Operations in Tensioning Steel Strands, by L. E. Hill. Early High-strength Concrete for Prestressing, by P. Klieger. Properties of Lightweight Concrete Related to Prestressing, by T. R. Jones, Jr., and H. K. Stephenson. Concrete for Prestressed and Reinforced Concrete Structures in the U.S.S.R., by B. G. Skramtaev. The Properties of Cement Paste for Grouting by Masaichi Okushima. Creep and Shrinkage of Solite Concretes, by G. L. Rogers. Inspection of Materials and Stressing Operations, by J. J. Waddell.

Bridges.—Construction Experience in California, by A. L. Elliott. The Bureau of Public Roads Criteria for Bridges, by E. L. Erickson. The 24-mile Lake Pontchartrain Bridge, by W. F. Palmer. Bridges for the Illinois Toll Highway, by M. E. Bender. Full-scale Test of Bridge on Northern Illinois Toll Highway, by J. Janney and W. J. Eney. Concentrated Tendons for Long-Span Bridges, by F. Leonhardt. Suspension Bridges, by D. C. C. Vandepitte. Development of Prestressed Concrete in New Zealand by the Ministry of Works, by B. W. Spooner and R. G. Norman. Bridges in the U.S.S.R., by I. Barenboim. Taipei Tamshui Bridge, by Wen-Tao Chang. Design, Erection, and Load Test of the Kamidai Bridge, by T. Nishina, M. Kimura, and M. Naruoka. Rectangular Bridge Deck Members in the United States, by S. L. Selvaggio and P. E. Balcomb. Alternate Bidding on Gorgas Lane Bridge, Philadelphia, by N. W. Willis.

Buildings.—Units for Reynolds Aluminium Plant, by Ross H. Bryan. Prestressed Lift-slabs, by C. Peterson and A. H. Brownfield. Prestressed Shells and Precast Units, by A. M. Haas. Fire Resistance of Prestressed Concrete, by A. W. Hill and L. A. Ashton. Prestressing in Italy, by Franco Levi. Rigid Building Frames of Post-tensioned Prestressed Concrete in Active Earthquake Area, by S. Ban. Prestressed Construction in Poland, by M. Rzedowski and J. Medwadowski. Potentialities for Long-span Structures in U.S.A., by D. P. Billington. Bauxite Storage Building in Jamaica, by C. J. Evans. Vibration-Resistant Foundations for Engines and Compressors, by S. H. Fistedis. A 20-metre Free Span Folded Prestressed Thin Shell Roof, by Florencio del Pozo.

Ground Slabs, Wharves, Piles, Dams, and Masts.—Airfield Pavements, by F. M. Mellinger. Piling and Marine Structures, by B. C. Gerwick, Jr., and W. J. Talbot, Jr. Cylinder Piles, by M. Fornerod. Applications of Prestressing to Dams, by J. Muller. Masts for Power Lines, by R. J. D. Finfrock.

Research.—Research at the University of Illinois, by C. P. Siess and M. A. Sozen. Research at Lehigh University, by C. E.

Ekberg, Jr. Calcium Chloride in Prestressed Concrete, by R. H. Evans. X-rays for Measuring Bond Stresses, by R. H. Evans and A. Williams. Diagonal Tension in Beams, by C. J. Bernhardt. Flexural Strength of Beams at Transfer, by A. C. Scordelis, T. Y. Lin, and Howard R. May. Anchorage-zone Stress Distribution, by S. Ban, H. Muguruma and Z. Ogaki. Loss of Tension Caused by Cable Friction, by C. Cestelli Guidi. Torsional Strength of Prestressed Concrete, by H. J. Cowan and S. Armstrong. Design and Proof Test of an Inverted Tee-Beam, by R. M. Gensert and J. B. Scalzi. Load Distribution Test—Sussex Street Bridge, Ottawa, Canada, by W. D. Houston and W. R. Schriever. Load Distribution in Model of Slab Bridge, by S. Inomata. Design Method of Statically-indeterminate Structures, by P. B. Morice. Stresses in End Blocks of Beams by Lattice Analogy, by G. S. Ramaswamy and H. Goel. Analysis of Multi-beam Bridges as Orthotropic Plates, by A. Roesli and R. Walther. Self-stressed Concrete, by V. V. Mikhailov. Suspended Roof for a Sports Stadium of 310 ft. Diameter, by M. Shupack.

Manufacture of Precast Prestressed Concrete.—Pretensioning Production Methods in U.S.A., by P. J. Verna, Jr. Sectional Precast Concrete Beams, by E. W. H. Gifford. Automation in Production of Prestressed Units in the U.S.S.R., by V. V. Mikhailov.

Design and Construction.—Recent Public Works in Pakistan, by K. Billig. Precast Concrete with High-tensile Bars, by D. H. Lee. Composite Construction in Great Britain, by F. J. Samuely. Two Structures in Yugoslavia, by B. Zezelj.

A Precast Blast-Screen.

A TRIAL length of a precast blast-screen (*Fig.* 1) for use at airports has recently been erected for the British Overseas Airways Corporation at London Airport. It is 109 ft. long, 13 ft. high, and leans toward the aircraft at an angle of 60 deg., so as to divert upwards the slipstream from the engines of Britannia aircraft standing about 60 ft. away. The screen comprises angular frames at 6 ft. 4 in. centres and bolted to a continuous concrete footing 2 ft. 6 in. square in cross section. The frames are 7½ in. wide and slots are cast in both sides to receive precast concrete planks 12 in. wide by 1¾ in. thick. The units were made and the screen was built by the Modular Concrete Co., Ltd.

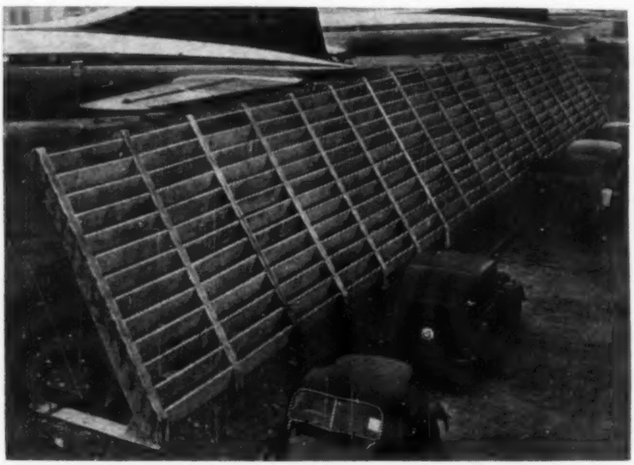


Fig. 1.

Effect of Frost on Mortar.

A SERIES of tests on the effect of freezing and thawing on mortars of various compositions has been made by Mr. Sven G. Bergström, and the results are published (in the Swedish language) in Bulletin No. 32 of the Swedish Cement and Concrete Research Institute, of Stockholm. The results of the tests may be summarised as follows.

(1) If the water-cement ratio, or the cement-paste content, or the grading, be varied in such a manner that the consistency of the mortar becomes more fluid, then the resistance to frost decreases. (2) If the water-cement ratio and the cement-paste content be varied at the same time, while the consistency remains unchanged, then the effect of the water-cement ratio is predominant. (3) It seems that the effect of the water-cement ratio and the cement-paste content can be represented by the product of these two quantities if the water-cement ratio is reduced by a certain definite value, in this case 0·20, corresponding to the proportions at which the quantity of freezable water is zero. It appears that the effect of these two factors can also be expressed by the water content during mixing, provided that this content is reduced in a similar manner. (4) The resistance to frost decreases as the water-absorbing capacity of the hardened mortar becomes greater. (5) If the specimens are cured in water continuously until they are tested, the resistance to frost first becomes greater with age, but then markedly decreases. (6) Excessive vibration caused a slight decrease in resistance to frost, but less vibration had widely varying results. The degree of compaction has no dominant effect on the resistance to frost of the mortar as a whole.

The methods of testing are described in the paper, and it is stated that autogenous healing during the period of thawing seems to have a very important influence on the rate of deterioration. Two non-destructive methods of testing were used for observation of changes in quality, namely, measurements of the change in length and measurements of the dynamic modulus of elasticity; the latter method is preferred. It is recommended that the modulus of elasticity should not be measured by means of supersonic waves, but may be determined, for example, by measuring the characteristic frequency of bending due to vibration.

The Bulletin is obtainable from the Institute at a price of 2 Kr.

FIFTY YEARS AGO.

From "CONCRETE AND CONSTRUCTIONAL ENGINEERING", MAY–JUNE, 1908.

QUEBEC BRIDGE DISASTER.—We have previously commented on the fact that the Quebec disaster, involving the loss of some 80 lives and £500,000 of money, called for but little attention in this country, whilst any accident involving perhaps the loss of only £500 with a reinforced concrete structure is generally made the subject of extensive denunciation and adverse criticism. The report of the Royal Commission appointed by the Canadian Government to investigate this terrible disaster has been issued. It is one of the most lucid and painstaking documents ever prepared in respect to an engineering disaster. There is not the least doubt that the contractor's design in this case was "cut" too finely, and likewise the consulting engineer had, if anything, a tendency to cut the quantity of the steel used, either for reasons of professional vanity or in the interests of the employer. Loads and dead weights were taken too finely, stresses and strains that should have been provided for were underestimated, and the result has been that both the designer for the contractor and the consulting engineer are held responsible for the disaster.

REINFORCED CONCRETE RAILWAY SLEEPERS.—We understand that the Italian State Railways, after experiments, have decided to employ sleepers of reinforced concrete on their lines, and a first lot of 300,000 sleepers have been ordered at a cost of about 6*s*. 4*d*. each. Experiments are also being made in Canada, particulars of which we hope to publish in an early issue.

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